



## RICCI FLOW ON MODIFIED RIEMANN EXTENSIONS

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Communicated by Bernard Kay

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**Abstract.** We study the properties of the modified Riemann extensions evolving under the Ricci flow with examples.

MSC: 53C20, 53C44

Keywords: Evolution equations, Ricci flow, Riemann extension

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### 1. Introduction

The Ricci flow and the evolution equations of the Riemannian curvature tensor were initially introduced by Hamilton [8] and was later studied to a large extent by Perelman [13–15], Cao and Zhu [4], Morgan and Tian [10]. Indeed, the theory of Ricci flow has been used to prove the geometrization and Poincaré conjectures [1]. However not much work has been done on Ricci flows on modified Riemann extensions. The Ricci flow equation is the evolution equation  $\frac{\partial g_{ij}}{\partial t} = -2R_{ij}$  where  $g_{ij}$  and  $R_{ij}$  are metric components respectively. As flow progresses the metric changes and hence the properties related to it.

Patterson and Walker [11] have defined Riemann extensions and showed how a Riemannian structure can be given to the  $2n$  dimensional tangent bundle of an  $n$ -dimensional manifold with given non-Riemannian structure. This shows that Riemann extension provides a solution of the general problem of embedding a manifold  $M$  carrying a given structure in a manifold  $\tilde{M}$  carrying another structure, the embedding being carried out in such a way that the structure on  $\tilde{M}$  induces in a natural way the given structure on  $M$ . The Riemann extension of Riemannian or non-Riemannian spaces can be constructed with the help of the Christoffel coefficients  $\Gamma^i_{jk}$  of corresponding Riemann space or with connection coefficients  $\Pi^i_{jk}$  in the case of the space of affine connection [5]. The theory of Riemann extensions has been extensively studied by Afifi [1]. Though the Riemann extensions itself is rich in geometry, here in our discussions, the modified Riemann extensions fit naturally in to the frame work. Modified Riemann extensions are introduced in [2] and their properties we list briefly in the next section.