



# FLAT AFFINE AND SYMPLECTIC GEOMETRIES ON LIE GROUPS

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**Abstract.** In this paper we exhibit a family of flat left invariant affine structures on the double Lie group of the oscillator Lie group of dimension 4, associated to each solution of classical Yang-Baxter equation given by Boucetta and Medina. On the other hand, using Koszul’s method, we prove the existence of an immersion of Lie groups between the group of affine transformations of a flat affine and simply connected manifold and the classical group of affine transformations of  $\mathbb{R}^n$ . In the last section, for each flat left invariant affine symplectic connection on the group of affine transformations of the real line, describe by Medina-Saldarriaga-Giraldo, we determine the affine symplectomorphisms. Finally we exhibit the Hess connection, associated to a Lagrangian bi-foliation, which is flat left invariant affine.

MSC: 53C07, 53B05, 53D05, 17B62, 17D25

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