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DEFORMATIONS OF THE VIRASORO ALGEBRA OF KRICHEVER – NOVIKOV TYPE

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Abstract. It is the general impression that deformation problems are always governed by cohomology spaces. In this contribution we consider the deformation of Lie algebras. There this close connection is true for finite-dimensional algebras, but fails for infinite dimensional ones. We construct geometric families of infinite dimensional Lie algebras over the moduli space of complex one-dimensional tori with marked points. These algebras are algebras of Krichever-Novikov type which consist of meromorphic vector fields of certain type over the tori. The families are non-trivial deformations of the (infinite dimensional) Witt algebra, and the Virasoro algebra respectively, despite the fact that the cohomology space associated to the deformation problem of the Witt algebra vanishes, and hence the algebra is formally rigid. A similar construction works for current algebras. The presented results are jointly obtained with Alice Fialowski.

1. Introduction

In this write-up of a talk, presented at the Białowieża meeting on "Geometric Methods in Physics" in 2005, I will give families of Lie algebras, which are non-trivial deformations of "formally rigid" infinite dimensional Lie algebras. The Lie algebras deformed are the Witt algebra, its universal central extension (i.e., the Virasoro algebra), the current algebras, and their central extensions (i.e., the affine Lie algebras). These algebras play an important role in Conformal Field Theory (CFT). The deformed algebras are of Krichever-Novikov type [7] and appear in particular in the context of a global operator approach to CFT [15], [16].

The algebras to be deformed are formally rigid, i.e., they only admit trivial deformations over the formal power series. Nevertheless, the constructed families are such that the deformations are locally non-trivial, where "locally" means that they are considered over small Zariski open or analytically open subsets of the deformation space containing the special point, corresponding to the algebra to be deformed. This phenomena is peculiar to, and in fact only possible, for infinite