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COMPLEXITY FOR INFINITE WORDS ASSOCIATED WITH QUADRATIC NON-SIMPLE PARRY NUMBERS*

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Abstract. Studying of complexity of infinite aperiodic words, i.e., the number of different factors of the infinite word of a fixed length, is an interesting combinatorial problem. Moreover, investigation of infinite words associated with β-integers can be interpreted as investigation of one-dimensional quasicrystals. In such a way of interpretation, complexity corresponds to the number of local configurations of atoms.

1. Introduction

To study the structure of an infinite word u on a finite alphabet $\mathcal{A}$ and to measure the diversity of patterns occurring in this word, it is useful to define complexity of u. It is a function $C(n)$ which with every $n \in \mathbb{N}$ associates the number of different words of length n contained in u. The simplest infinite word is a constant sequence z^ω with $z \in \mathcal{A}$. There exists only one word of each length, therefore $C(n) = 1$ for all $n \in \mathbb{N}$. One extreme of the opposite side is a random sequence for which, almost surely, the complexity $C(n) = (\#\mathcal{A})^n$. Between these two extremes, one can find infinite eventually periodic words for which the complexity $C(n) \leq n$ for all $n \in \mathbb{N}$, and the simplest aperiodic words, called *Sturmian words*, with the complexity $C(n) = n + 1$ for all $n \in \mathbb{N}$.

Some kinds of infinite aperiodic words can serve as models for one dimensional quasicrystals, i.e., materials with long-range orientational order and sharp diffraction images of non-crystallographic symmetry. To understand the physical properties of these materials, it is important to describe their combinatorial properties. For instance, complexity corresponds to the number of local configurations of atoms.

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