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ON THE  $\ell_p$  NORM OF GCD AND RELATED MATRICES

PENTTI HAUKKANEN

Department of Mathematics, Statistics and Philosophy,  
FIN-33014 University of Tampere, Finland

EMail: [mapehau@uta.fi](mailto:mapehau@uta.fi)

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## Abstract

We estimate the  $\ell_p$  norm of the  $n \times n$  matrix, whose  $ij$  entry is  $(i, j)^s/[i, j]^r$ , where  $r, s \in \mathbb{R}$ ,  $(i, j)$  is the greatest common divisor of  $i$  and  $j$  and  $[i, j]$  is the least common multiple of  $i$  and  $j$ .

*2000 Mathematics Subject Classification:* 11C20; 15A36; 11A25

*Key words:* GCD matrix, LCM matrix, Smith's determinant,  $\ell_p$  norm,  $O$ -estimate

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# 1. Introduction

Let  $S = \{x_1, x_2, \dots, x_n\}$  be a set of distinct positive integers, and let  $f$  be an arithmetical function. Let  $(S)_f$  denote the  $n \times n$  matrix having  $f$  evaluated at the greatest common divisor  $(x_i, x_j)$  of  $x_i$  and  $x_j$  as its  $ij$  entry, that is,  $(S)_f = (f((x_i, x_j)))$ . Analogously, let  $[S]_f$  denote the  $n \times n$  matrix having  $f$  evaluated at the least common multiple  $[x_i, x_j]$  of  $x_i$  and  $x_j$  as its  $ij$  entry, that is,  $[S]_f = (f([x_i, x_j]))$ . The matrices  $(S)_f$  and  $[S]_f$  are referred to as the GCD and LCM matrix on  $S$  associated with  $f$ , respectively. H. J. S. Smith [7] calculated  $\det(S)_f$  when  $S$  is a factor-closed set and  $\det[S]_f$  in a more special case. Since Smith, a large number of results on GCD and LCM matrices have been presented in the literature. For general accounts see e.g. [3, 4, 5].

Norms of GCD matrices have not been studied much in the literature. Some results are obtained in [2, 8, 9, 10, 11]. In this paper we provide further results.

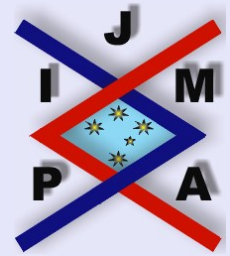
Let  $p \in \mathbb{Z}^+$ . The  $\ell_p$  norm of an  $n \times n$  matrix  $M$  is defined as

$$\|M\|_p = \left( \sum_{i=1}^n \sum_{j=1}^n |m_{ij}|^p \right)^{\frac{1}{p}}.$$

Let  $r, s \in \mathbb{R}$ . Let  $A$  denote the  $n \times n$  matrix, whose  $i, j$  entry is given as

$$(1.1) \quad a_{ij} = \frac{(i, j)^s}{[i, j]^r},$$

where  $(i, j)$  is the greatest common divisor of  $i$  and  $j$  and  $[i, j]$  is the least common multiple of  $i$  and  $j$ . For  $s = 1, r = 0$  and  $s = 0, r = -1$ , respectively, the matrix  $A$  is the GCD and the LCM matrix on  $\{1, 2, \dots, n\}$ . For  $s = 1, r = 1$



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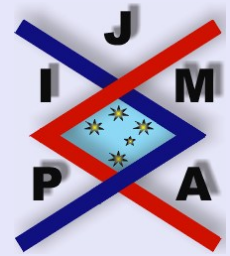
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the matrix  $A$  is the Hadamard product of the GCD matrix and the reciprocal LCM matrix on  $\{1, 2, \dots, n\}$ . In this paper we estimate the  $\ell_p$  norm of the matrix  $A$  given in (1.1) for all  $r, s \in \mathbb{R}$  and  $p \in \mathbb{Z}^+$ .



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## 2. Preliminaries

In this section we review the basic results on arithmetical functions needed in this paper. For more comprehensive treatments on arithmetical functions and their estimates we refer to [1] and [6].

The Dirichlet convolution  $f * g$  of two arithmetical functions  $f$  and  $g$  is defined as

$$(f * g)(n) = \sum_{d|n} f(d)g(n/d).$$

Let  $N^u$ ,  $u \in \mathbb{R}$ , denote the arithmetical function defined as  $N^u(n) = n^u$  for all  $n \in \mathbb{Z}^+$ , and let  $E$  denote the arithmetical function defined as  $E(n) = 1$  for all  $n \in \mathbb{Z}^+$ . The divisor function  $\sigma_u$ ,  $u \in \mathbb{R}$ , is defined as

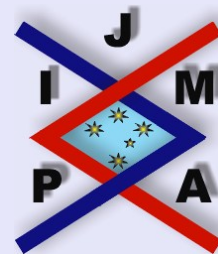
$$(2.1) \quad \sigma_u(n) = \sum_{d|n} d^u = (N^u * E)(n).$$

The Jordan totient function  $J_k(n)$ ,  $k \in \mathbb{Z}^+$ , is defined as the number of  $k$ -tuples  $a_1, a_2, \dots, a_k \pmod{n}$  such that the greatest common divisor of  $a_1, a_2, \dots, a_k$  and  $n$  is 1. By convention,  $J_k(1) = 1$ . The Möbius function  $\mu$  is the inverse of  $E$  under the Dirichlet convolution. It is well known that  $J_k = N^k * \mu$ . This suggests we define  $J_u = N^u * \mu$  for all  $u \in \mathbb{R}$ . Since  $\mu$  is the inverse of  $E$  under the Dirichlet convolution, we have

$$(2.2) \quad n^u = \sum_{d|n} J_u(d).$$

It is easy to see that

$$J_u(n) = n^u \prod_{p|n} (1 - p^{-u}).$$



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We thus have

$$(2.3) \quad 0 \leq J_u(n) \leq n^u \text{ for } u \geq 0.$$

**Lemma 2.1.**

(a) If  $s > -1$ , then  $\sum_{k \leq n} k^s = O(n^{s+1})$ .

(b)  $\sum_{k \leq n} k^{-1} = O(\log n)$ .

(c) If  $s < -1$ , then  $\sum_{k \leq n} k^s = O(1)$ .

Lemma 2.1 follows from the well-known asymptotic formulas for  $n^s$ , see [1, Chapter 3].

**Lemma 2.2.** Suppose that  $t > 1$ .

(a) If  $u - t > -1$ , then  $\sum_{k \leq n} \frac{\sigma_u(k)}{k^t} = O(n^{u-t+1})$ .

(b) If  $u - t = -1$ , then  $\sum_{k \leq n} \frac{\sigma_u(k)}{k^t} = O(\log n)$ .

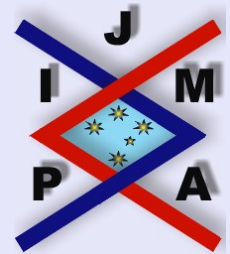
(b) If  $u - t < -1$ , then  $\sum_{k \leq n} \frac{\sigma_u(k)}{k^t} = O(1)$ .

*Proof.* For all  $u$  and  $t$  we have

$$(2.4) \quad \sum_{k \leq n} \frac{\sigma_u(k)}{k^t} = \sum_{k \leq n} k^{-t} \sum_{d|k} d^u = \sum_{d \leq n} d^{u-t} q^{-t} = \sum_{d \leq n} d^{u-t} \sum_{q \leq n/d} q^{-t}.$$

Now, let  $t > 1$ . Then, by Lemma 2.1(c),

$$\sum_{k \leq n} \frac{\sigma_u(k)}{k^t} = O(1) \sum_{d \leq n} d^{u-t}.$$



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If  $u - t > -1$ , then on the basis of Lemma 2.1(a)

$$\sum_{k \leq n} \frac{\sigma_u(k)}{k^t} = O(1)O(n^{u-t+1}).$$

If  $u - t = -1$ , then on the basis of Lemma 2.1(b)

$$\sum_{k \leq n} \frac{\sigma_u(k)}{k^t} = O(1)O(\log n).$$

If  $u - t < -1$ , then on the basis of Lemma 2.1(c)

$$\sum_{k \leq n} \frac{\sigma_u(k)}{k^t} = O(1)O(1).$$

□

### Lemma 2.3.

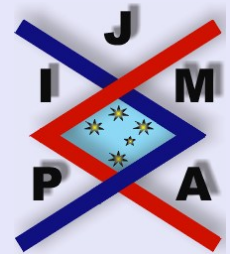
(a) If  $u > 0$ , then  $\sum_{k \leq n} \frac{\sigma_u(k)}{k} = O(n^u \log n)$ .

(b) If  $u = 0$ , then  $\sum_{k \leq n} \frac{\sigma_u(k)}{k} = O(\log^2 n)$ .

(c) If  $u < 0$ , then  $\sum_{k \leq n} \frac{\sigma_u(k)}{k} = O(\log n)$ .

*Proof.* According to (2.4) with  $t = 1$  we have

$$\sum_{k \leq n} \frac{\sigma_u(k)}{k} = \sum_{d \leq n} d^{u-1} \sum_{q \leq n/d} q^{-1}.$$



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Thus on the basis of Lemma 2.1(b)

$$\sum_{k \leq n} \frac{\sigma_u(k)}{k^t} = O(\log n) \sum_{d \leq n} d^{u-1}.$$

If  $u > 0$ , then on the basis of Lemma 2.1(a)

$$\sum_{k \leq n} \frac{\sigma_u(k)}{k^t} = O(\log n) O(n^u).$$

If  $u = 0$ , then on the basis of Lemma 2.1(b)

$$\sum_{k \leq n} \frac{\sigma_u(k)}{k^t} = O(\log n) O(\log n).$$

If  $u < 0$ , then on the basis of Lemma 2.1(c)

$$\sum_{k \leq n} \frac{\sigma_u(k)}{k^t} = O(\log n) O(1).$$

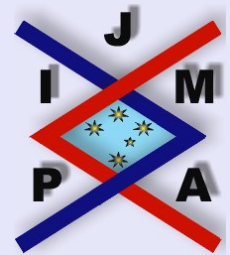
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**Lemma 2.4.** Suppose that  $t < 1$ .

(a) If  $u > 0$ , then  $\sum_{k \leq n} \frac{\sigma_u(k)}{k^t} = O(n^{1+u-t})$ .

(b) If  $u = 0$ , then  $\sum_{k \leq n} \frac{\sigma_u(k)}{k^t} = O(n^{1-t} \log n)$ .

(c) If  $u < 0$ , then  $\sum_{k \leq n} \frac{\sigma_u(k)}{k^t} = O(n^{1-t})$ .



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*Proof.* According to (2.4) we have

$$\sum_{k \leq n} \frac{\sigma_u(k)}{k^t} = \sum_{d \leq n} d^{u-t} \sum_{q \leq n/d} q^{-t}.$$

Thus on the basis of Lemma 2.1(a)

$$\sum_{k \leq n} \frac{\sigma_u(k)}{k^t} = O(n^{1-t}) \sum_{d \leq n} d^{u-1}.$$

If  $u > 0$ , then on the basis of Lemma 2.1(a)

$$\sum_{k \leq n} \frac{\sigma_u(k)}{k^t} = O(n^{1-t})O(n^u).$$

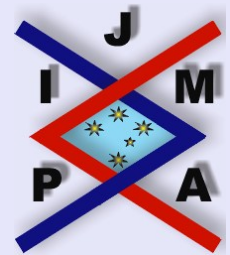
If  $u = 0$ , then on the basis of Lemma 2.1(b)

$$\sum_{k \leq n} \frac{\sigma_u(k)}{k^t} = O(n^{1-t})O(\log n).$$

If  $u < 0$ , then on the basis of Lemma 2.1(c)

$$\sum_{k \leq n} \frac{\sigma_u(k)}{k^t} = O(n^{1-t})O(1).$$

□



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### 3. Results

In Theorems 3.1, 3.2 and 3.3 we estimate the  $\ell_p$  norm of the matrix  $A$  given in (1.1). Their proofs are based on the formulas in Section 2 and the following observations.

Since  $(i, j)[i, j] = ij$ , we have for all  $p, r, s$

$$(3.1) \quad \|A\|_p^p = \sum_{i=1}^n \sum_{j=1}^n \frac{(i, j)^{sp}}{[i, j]^{rp}} = \sum_{i=1}^n \sum_{j=1}^n \frac{(i, j)^{(r+s)p}}{i^{rp} j^{rp}}.$$

From (2.2) we obtain

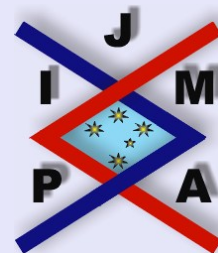
$$(3.2) \quad \begin{aligned} \|A\|_p^p &= \sum_{i=1}^n \frac{1}{i^{rp}} \sum_{j=1}^n \frac{1}{j^{rp}} \sum_{d|(i,j)} J_{(r+s)p}(d) \\ &= \sum_{i=1}^n \frac{1}{i^{rp}} \sum_{d|i} J_{(r+s)p}(d) \sum_{\substack{j=1 \\ d|j}}^n \frac{1}{j^{rp}} \\ &= \sum_{i=1}^n \frac{1}{i^{rp}} \sum_{d|i} \frac{J_{(r+s)p}(d)}{d^{rp}} \sum_{j=1}^{\lfloor n/d \rfloor} \frac{1}{j^{rp}}. \end{aligned}$$

**Theorem 3.1.** *Suppose that  $r > 1/p$ .*

(1) *If  $s > r - 1/p$ , then  $\|A\|_p = O(n^{s-r+1/p})$ .*

(2) *If  $s = r - 1/p$ , then  $\|A\|_p = O(\log^{1/p} n)$ .*

(3) *If  $s < r - 1/p$ , then  $\|A\|_p = O(1)$ .*



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*Proof.* Let  $r > 1/p$  or  $rp > 1$ . Then, by (3.2) and Lemma 2.1(c),

$$\|A\|_p^p = O(1) \sum_{i=1}^n \frac{1}{i^{rp}} \sum_{d|i} \frac{|J_{(r+s)p}(d)|}{d^{rp}}.$$

Assume that  $r + s \geq 0$ . Then, by (2.3) and (2.1),

$$\|A\|_p^p = O(1) \sum_{i=1}^n \frac{\sigma_{sp}(i)}{i^{rp}}.$$

**Case 1.** Let  $s > r - 1/p$  or  $sp - rp > -1$ . Then, by Lemma 2.2(a),

$$\|A\|_p^p = O(1)O(n^{sp-rp+1}) = O(n^{sp-rp+1}).$$

**Case 2.** Let  $s = r - 1/p$  or  $sp - rp = -1$ . Then, by Lemma 2.2(b),

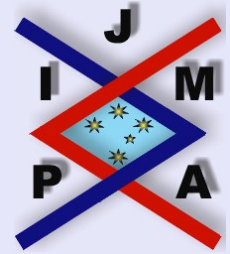
$$\|A\|_p^p = O(1)O(\log n) = O(\log n).$$

**Case 3.** Let  $s < r - 1/p$  or  $sp - rp < -1$ . Then, by Lemma 2.2(c),

$$\|A\|_p^p = O(1)O(1) = O(1).$$

Now, assume that  $r + s < 0$ . Since  $r > 1/p$ , we have  $s < r - 1/p$  and thus we consider Case 3. Since  $r + s < 0$ , then  $(i, j)^{(r+s)p} \leq 1$  and thus on the basis of (3.1) we have

$$\|A\|_p^p \leq \sum_{i=1}^n i^{-rp} \sum_{j=1}^n j^{-rp}.$$



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Since  $rp > 1$ , we obtain from Lemma 2.1(c)

$$\|A\|_p^p = O(1)O(1) = O(1).$$

□

**Theorem 3.2.** *Suppose that  $r = 1/p$ .*

(1) *If  $s > 0$ , then  $\|A\|_p = O(n^s \log^{2/p} n)$ .*

(2) *If  $s = 0$ , then  $\|A\|_p = O(\log^{3/p} n)$ .*

(3) *If  $s < 0$ , then  $\|A\|_p = O(\log^{2/p} n)$ .*

*Proof.* From (3.2) with  $rp = 1$  we obtain

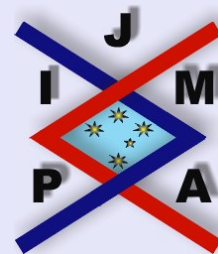
$$\|A\|_p^p = \sum_{i=1}^n \frac{1}{i} \sum_{d|i} \frac{J_{sp+1}(d)}{d} \sum_{j=1}^{[n/d]} \frac{1}{j}.$$

By Lemma 2.1(b),

$$\|A\|_p^p = O(\log n) \sum_{i=1}^n \frac{1}{i} \sum_{d|i} \frac{|J_{sp+1}(d)|}{d}.$$

Assume that  $sp + 1 \geq 0$ . Then, by (2.3) and (2.1),

$$\|A\|_p^p = O(\log n) \sum_{i=1}^n \frac{\sigma_{sp}(i)}{i}.$$



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**Case 4.** Assume that  $s > 0$  or  $sp > 0$ . Then, by Lemma 2.3(a),

$$\|A\|_p^p = O(\log n)O(n^{sp} \log n) = O(n^{sp} \log^2 n).$$

**Case 5.** Assume that  $s = 0$  or  $sp = 0$ . Then, by Lemma 2.3(b),

$$\|A\|_p^p = O(\log n)O(\log^2 n) = O(\log^3 n).$$

**Case 6.** Assume that  $s < 0$  or  $sp < 0$ . Then, by Lemma 2.3(c),

$$\|A\|_p^p = O(\log n)O(\log n) = O(\log^2 n).$$

Now, assume that  $sp + 1 < 0$ . Then  $s < 0$  and thus we consider Case 6. Since  $sp + 1 < 0$  and  $rp = 1$ , then  $(i, j)^{(r+s)p} \leq 1$  and thus on the basis of (3.1) we have

$$\|A\|_p^p \leq \sum_{i=1}^n i^{-rp} \sum_{j=1}^n j^{-rp}.$$

Since  $rp = 1$ , we obtain from Lemma 2.1(b)

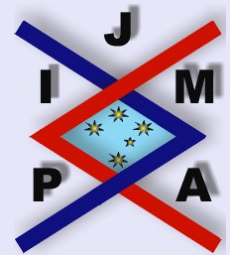
$$\|A\|_p^p = O(\log n)O(\log n) = O(\log^2 n).$$

□

**Theorem 3.3.** Suppose that  $r < 1/p$ .

(1) If  $s > -r + 1/p$ , then  $\|A\|_p = O(n^{s-r+1/p})$ .

(2) If  $s = -r + 1/p$ , then  $\|A\|_p = O(n^{-2r+2/p} \log^{1/p} n)$ .



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(3) If  $s < -r + 1/p$ , then  $\|A\|_p = O(n^{-2r+2/p})$ .

*Proof.* Let  $r < 1/p$  or  $rp < 1$ . By (3.2) and Lemma 2.1(a),

$$\|A\|_p^p = O(n^{1-rp}) \sum_{i=1}^n \frac{1}{i^{rp}} \sum_{d|i} \frac{|J_{(r+s)p}(d)|}{d}.$$

Assume that  $r + s \geq 0$ . Then, by (2.3) and (2.1),

$$\|A\|_p^p = O(n^{1-rp}) \sum_{i=1}^n \frac{\sigma_{(r+s)p-1}(i)}{i^{rp}}.$$

**Case 7.** Let  $s > -r + 1/p$  or  $(r + s)p - 1 > 0$ . Then, by Lemma 2.4(a),

$$\|A\|_p^p = O(n^{1-rp}) O(n^{1+(r+s)p-1-rp}) = O(n^{1+sp-rp}).$$

**Case 8.** Let  $s = -r + 1/p$  or  $(r + s)p - 1 = 0$ . Then, by Lemma 2.4(b),

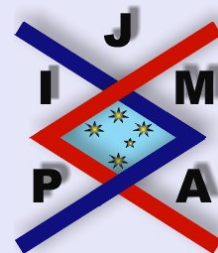
$$\|A\|_p^p = O(n^{1-rp}) O(n^{1-rp} \log n) = O(n^{2-2rp} \log n).$$

**Case 9.** Let  $s < -r + 1/p$  or  $(r + s)p - 1 < 0$ . Then, by Lemma 2.4(c),

$$\|A\|_p^p = O(n^{1-rp}) O(n^{1-rp}) = O(n^{2-2rp}).$$

Now, assume that  $r + s < 0$ . Then  $s < -r + 1/p$  and thus we consider Case 9. Since  $r + s < 0$ , then  $(i, j)^{(r+s)p} \leq 1$  and thus on the basis of (3.1) we have

$$\|A\|_p^p \leq \sum_{i=1}^n i^{-rp} \sum_{j=1}^n j^{-rp}.$$



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Since  $rp < 1$ , we obtain from Lemma 2.1(a)

$$\|A\|_p^p = O(n^{1-rp})O(n^{1-rp}) = O(n^{2-2rp}).$$

□

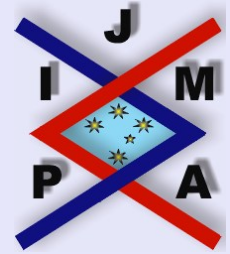
### Corollary 3.4.

- (a)  $\|(i, j)\|_p = O(n^{1+1/p})$  when  $p \geq 2$ .
- (b)  $\|(i, j)\|_p = O(n^2 \log n)$  when  $p = 1$ .
- (c)  $\|[i, j]\|_p = O(n^{2+2/p})$  when  $p \geq 1$ .
- (d)  $\|(i, j)/[i, j]\|_p = O(n^{1/p})$  when  $p \geq 2$ .
- (e)  $\|(i, j)/[i, j]\|_p = O(n \log^2 n)$  when  $p = 1$ .

*Proof.*

- (a) Take  $r = 0, s = 1, p \geq 2$  in Case 7 of Theorem 3.3.
- (b) Take  $r = 0, s = 1, p = 1$  in Case 8 of Theorem 3.3.
- (c) Take  $r = -1, s = 0, p \geq 1$  in Case 9 of Theorem 3.3.
- (d) Take  $r = s = 1, p \geq 2$  in Case 1 of Theorem 3.1.
- (e) Take  $r = s = p = 1$  in Case 4 of Theorem 3.2.

□



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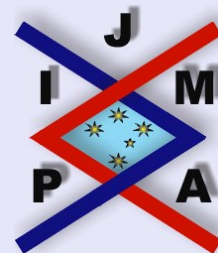
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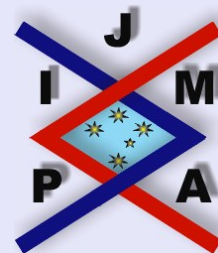
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