

THE REFINEMENT AND REVERSE OF A GEOMETRIC INEQUALITY

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Abstract: In this paper, we give a refinement and a reverse of a geometric inequality in a triangle posed by Jiang [2] by making use of the equivalent form of a fundamental inequality [6] and classic analysis.

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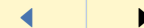
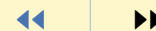
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1. Introduction and Main Result

For $\triangle ABC$, let a, b, c be the side-lengths, A, B, C the angles, s the semi-perimeter, R the circumradius and r the inradius, respectively. Moreover, we will customarily use the cyclic sum symbols, that is: $\sum f(a) = f(a) + f(b) + f(c)$, $\sum f(b, c) = f(a, b) + f(b, c) + f(c, a)$ and $\prod f(a) = f(a)f(b)f(c)$ etc.

In 2008, Jiang [2] posed the following geometric inequality problem.

Problem 1. In $\triangle ABC$, prove that

$$(1.1) \quad \sum \frac{a}{b+c} \left(\tan^4 \frac{B}{2} + \tan^4 \frac{C}{2} \right) \geq \frac{1}{3}.$$

In the same year, Manh Dung Nguyen and Duy Khanh Nguyen [4] proved inequality (1.1).

In this paper, we give a refinement and a reverse of inequality (1.1).

Theorem 1.1. In $\triangle ABC$, the best constant k for the following inequality

$$(1.2) \quad \sum \frac{a}{b+c} \left(\tan^4 \frac{B}{2} + \tan^4 \frac{C}{2} \right) \geq \frac{1}{3} + k \left(1 - \frac{2r}{R} \right).$$

is $\lambda_0 \approx 1.330090721$ which is the positive real root of:

$$(1.3) \quad 3564\lambda^6 + 114588\lambda^5 - 246261\lambda^4 + 137484\lambda^3 - 29712\lambda^2 + 2336\lambda - 60 = 0$$

It is easy to see that inequality (1.1) follows from Theorem 1.1 and Euler's inequality $R \geq 2r$ immediately.

Theorem 1.2. In $\triangle ABC$, we have

$$(1.4) \quad \sum \frac{a}{b+c} \left(\tan^4 \frac{B}{2} + \tan^4 \frac{C}{2} \right) \leq \frac{1}{3} + \frac{8}{3} \left[\left(\frac{R}{2r} \right)^2 - 1 \right].$$



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2. Preliminary Results

In order to prove Theorem 1.1 and Theorem 1.2, we shall require the following five lemmas.

Lemma 2.1 ([6]). *For any triangle ABC , the following inequalities hold true:*

$$(2.1) \quad \frac{1}{4}\delta(4-\delta)^3 \leq \frac{s^2}{R^2} \leq \frac{1}{4}(2-\delta)(2+\delta)^3,$$

where $\delta = 1 - \sqrt{1 - \frac{2r}{R}} \in (0, 1]$. Equality on the left hand side of the double inequality (2.1) is valid if and only if triangle ABC is an isosceles triangle with top-angle greater than or equal to $\frac{\pi}{3}$, and equality on the right hand side of the double inequality (2.1) is valid if and only if triangle ABC is an isosceles triangle with top-angle less than or equal to $\frac{\pi}{3}$.

Lemma 2.2. *In $\triangle ABC$, we have*

$$(2.2) \quad \sum \frac{a}{b+c} \left(\tan^4 \frac{B}{2} + \tan^4 \frac{C}{2} \right) \\ = \frac{1}{s^4(s^2 + 2Rr + r^2)} \cdot [2s^6 - 2(32R^2 + 24Rr + r^2)s^4 \\ + 2(4R + r)(32R^3 + 72R^2r + 28Rr^2 + r^3)s^2 - 2r(2R + r)(4R + r)^4].$$

Proof. From the law of cosines, we get

$$\tan^2 \frac{A}{2} = \frac{\sin^2 \frac{A}{2}}{\cos^2 \frac{A}{2}} = \frac{1 - \cos A}{1 + \cos A} = \frac{1 - \frac{b^2+c^2-a^2}{2bc}}{1 + \frac{b^2+c^2-a^2}{2bc}} = \frac{(a+b-c)(c+a-b)}{(a+b+c)(b+c-a)}.$$



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In the same manner, we can also obtain

$$\tan^2 \frac{B}{2} = \frac{(b+c-a)(a+b-c)}{(a+b+c)(c+a-b)}, \quad \tan^2 \frac{C}{2} = \frac{(c+a-b)(b+c-a)}{(a+b+c)(a+b-c)}.$$

Hence,

$$\begin{aligned} (2.3) \quad & \sum \frac{a}{b+c} \left(\tan^4 \frac{B}{2} + \tan^4 \frac{C}{2} \right) \\ &= \sum \frac{a}{b+c} \left[\frac{(b+c-a)^2(a+b-c)^2}{(a+b+c)^2(c+a-b)^2} + \frac{(c+a-b)^2(b+c-a)^2}{(a+b+c)^2(a+b-c)^2} \right] \\ &= \frac{\sum a(c+a)(a+b)(b+c-a)^4[(a+b-c)^4 + (c+a-b)^4]}{(a+b+c)^2 \cdot \prod (b+c-a)^2 \cdot \prod (b+c)}. \end{aligned}$$

And it is not difficult to verify the following three identities.

$$(2.4) \quad \prod (b+c) = (ab+bc+ca)(a+b+c) - abc,$$

$$(2.5) \quad \prod (b+c-a) = -(a+b+c)^3 + 4(ab+bc+ca)(a+b+c) - 8abc,$$

$$\begin{aligned} (2.6) \quad & \sum a(c+a)(a+b)(b+c-a)^4[(a+b-c)^4 + (c+a-b)^4] \\ &= 2(a+b+c)^{11} - 28(ab+bc+ca)(a+b+c)^9 \\ &\quad - 18abc(a+b+c)^8 + 160(ab+bc+ca)^2(a+b+c)^7 \\ &\quad + 224abc(ab+bc+ca)(a+b+c)^6 \\ &\quad - 400a^2b^2c^2(a+b+c)^5 - 480(ab+bc+ca)^3(a+b+c)^5 \\ &\quad - 768abc(ab+bc+ca)^2(a+b+c)^4 \end{aligned}$$

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$$\begin{aligned} &+ 2560a^2b^2c^2(ab + bc + ca)(a + b + c)^3 \\ &+ 768(ab + bc + ca)^4(a + b + c)^3 - 1280a^3b^3c^3(a + b + c)^2 \\ &+ 512abc(ab + bc + ca)^3(a + b + c)^2 \\ &- 512(ab + bc + ca)^5(a + b + c) \\ &- 3328a^2b^2c^2(ab + bc + ca)^2(a + b + c) + 2048a^3b^3c^3(ab + bc + ca) \\ &+ 512abc(ab + bc + ca)^4, \end{aligned}$$

Identity (2.2) follows directly from identities (2.3) – (2.6) and the following known identities:

$$a + b + c = 2s, \quad ab + bc + ca = s^2 + 4Rr + r^2, \quad abc = 4Rrs.$$

□

Lemma 2.3 ([9]). In $\triangle ABC$, we have

$$(2.7) \quad s^4 - (20Rr - r^2)s^2 + 4r^2(4R + r)^2 \geq 0.$$

Lemma 2.4. The function

$$\begin{aligned} f(s) = &\frac{1}{s^4(s^2 + 2Rr + r^2)} \cdot [2s^6 - 2(32R^2 + 24Rr + r^2)s^4 \\ &+ 2(4R + r)(32R^3 + 72R^2r + 28Rr^2 + r^3)s^2 - 2r(2R + r)(4R + r)^4] \end{aligned}$$

is strictly monotone decreasing on the interval $[s_1, s_2]$, where

$$\begin{aligned} s_1 &= \sqrt{2R^2 + 10Rr - r^2 - 2(R - 2r)\sqrt{R^2 - 2Rr}} \\ &= \frac{1}{2}\sqrt{\delta(4 - \delta)^3}R \end{aligned}$$

and

$$\begin{aligned} s_2 &= \sqrt{2R^2 + 10Rr - r^2 + 2(R - 2r)\sqrt{R^2 - 2Rr}} \\ &= \frac{1}{2}\sqrt{(2 - \delta)(2 + \delta)^3 R}. \end{aligned}$$

Proof. Calculating the derivative for $f(s)$, we get

$$\begin{aligned} f'(s) &= \frac{-8}{s^5(s^2 + 2Rr + r^2)^2} \cdot \{(16R^2 + 13Rr + r^2)[s^4 - (20Rr - r^2)s^2 + 4r^2(4R + r)^2] \\ &\quad \cdot (4R^2 + 4Rr + 3r^2 - s^2) + 64R^4[s^4 - (20Rr - r^2)s^2 + 4r^2(4R + r)^2] \\ &\quad + (116R^3r + 164R^2r^2 + 18Rr^3 + r^4)[-s^4 + (4R^2 + 20Rr - 2r^2)s^2 \\ &\quad - r(4R + r)^3] + [1024488r^6 + 3177399r^5(R - 2r) + 4148540r^4(R - 2r)^2 \\ &\quad + 2913136r^3(R - 2r)^3 + 1156192r^2(R - 2r)^4 + 244816r(R - 2r)^5 \\ &\quad + 21504(R - 2r)^6]r^2\}. \end{aligned}$$

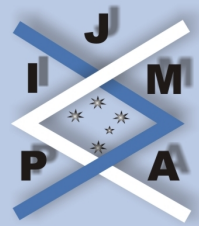
From inequality (2.7), **Euler's** inequality $R \geq 2r$, **Gerretsen's** inequality (see [1, page 45]) $s^2 \leq 4R^4 + 4Rr + 3r^2$ and the fundamental inequality (see [3, page 2])

$$-s^4 + (4R^2 + 20Rr - 2r^2)s^2 - r(4R + r)^3 \geq 0,$$

we can conclude that $f'(s) < 0$. Therefore, $f(s)$ is strictly monotone decreasing on the interval $[s_1, s_2]$. \square

Lemma 2.5 ([10]). Denote

$$\begin{aligned} f(x) &= a_0x^n + a_1x^{n-1} + \dots + a_n, \\ g(x) &= b_0x^m + b_1x^{m-1} + \dots + b_m. \end{aligned}$$



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If $a_0 \neq 0$ or $b_0 \neq 0$, then the polynomials $f(x)$ and $g(x)$ have common roots if and only if

$$R(f, g) = \begin{vmatrix} a_0 & a_1 & a_2 & \cdots & a_n & 0 & \cdots & 0 \\ 0 & a_0 & a_1 & \cdots & a_{n-1} & a_n & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & a_0 & \cdots & \cdots & \cdots & a_n \\ b_0 & b_1 & b_2 & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & b_0 & b_1 & \cdots & \cdots & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & b_0 & b_1 & \cdots & b_m \end{vmatrix} = 0,$$

where $R(f, g)$ is the Sylvester Resultant of $f(x)$ and $g(x)$.



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3. The Proof of Theorem 1.1

Proof. By Lemma 2.2 and Lemma 2.4, we get

$$(3.1) \quad \sum \frac{a}{b+c} \left(\tan^4 \frac{B}{2} + \tan^4 \frac{C}{2} \right) \\ \geq f(s_2) = \frac{\delta^6 - 7\delta^5 + 20\delta^4 - 24\delta^3 + 32\delta^2 - 48\delta + 32}{(\delta + 1)(\delta - 2)^2(2 + \delta)^2}.$$

Now we consider the best constant for the following inequality.

$$(3.2) \quad \frac{\delta^6 - 7\delta^5 + 20\delta^4 - 24\delta^3 + 32\delta^2 - 48\delta + 32}{(\delta + 1)(\delta - 2)^2(2 + \delta)^2} \\ \geq \frac{1}{3} + k \left(1 - \frac{2r}{R} \right) = \frac{1}{3} + k(1 - \delta)^2 \quad (0 < \delta \leq 1).$$

(i) In the case of $\delta = 1$, the inequality (3.2) obviously holds.

(ii) In the case of $0 < \delta < 1$, the inequality (3.2) is equivalent to

$$k \leq g(\delta) := \frac{3\delta^4 - 16\delta^3 + 24\delta^2 + 80}{3(\delta + 1)(\delta - 2)^2(\delta + 2)^2} \quad (0 < \delta < 1).$$

Calculating the derivative for $g(\delta)$, we get

$$g'(\delta) = \frac{3\delta^6 - 32\delta^5 + 92\delta^4 - 32\delta^3 + 304\delta^2 + 512\delta - 320}{3(\delta + 1)^2(2 - \delta)^3(\delta + 2)^3}.$$

Letting $g'(\delta) = 0$, we get

$$(3.3) \quad 3\delta^6 - 32\delta^5 + 92\delta^4 - 32\delta^3 + 304\delta^2 + 512\delta - 320 = 0, \quad (0 < \delta < 1).$$



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It is not difficult to see that the equation (3.3) has only one positive root on the open interval $(0, 1)$. Denote δ_0 to be the root of the equation (3.3). Then

$$(3.4) \quad g(\delta)_{min} = g(\delta_0) = \frac{3\delta_0^4 - 16\delta_0^3 + 24\delta_0^2 + 80}{(\delta_0 + 1)(\delta_0 - 2)^2(\delta_0 + 2)^2}.$$

It is easy to see that $g(\delta_0)$ is a root of the following nonlinear algebraic equation system.

$$(3.5) \quad \begin{cases} F(\delta_0) = 0, \\ G(\delta_0) = 0, \end{cases}$$

where

$$F(\delta_0) = 3(\delta_0 + 1)(\delta_0 - 2)^2(\delta_0 + 2)^2\lambda - (3\delta_0^4 - 16\delta_0^3 + 24\delta_0^2 + 80)$$

and

$$G(\delta_0) = 3\delta_0^6 - 32\delta_0^5 + 92\delta_0^4 - 32\delta_0^3 + 304\delta_0^2 + 512\delta_0 - 320.$$

Then,

$$R_{\delta_0}(F, G) = \begin{vmatrix} 3\lambda & 16 - 24\lambda & 3\lambda - 3 & \cdots & 48\lambda - 80 & 0 & \cdots & 0 \\ 0 & 3\lambda & 16 - 24\lambda & \cdots & 48\lambda & 48\lambda - 80 & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 3\lambda & \cdots & \cdots & \cdots & 48\lambda - 80 \\ 3 & -32 & 92 & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & 3 & -32 & \cdots & \cdots & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 3 & -32 & \cdots & -320 \end{vmatrix}$$

$$= -3177213868376064(3564\lambda^6 + 114588\lambda^5 - 246261\lambda^4 + 137484\lambda^3 - 29712\lambda^2 + 2336\lambda - 60).$$



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With Lemma 2.5, we can conclude that $g(\delta_0)$ is the real root of (1.3). And the equation (1.3) has only one positive real root, hence, $g(\delta_0)$ is the positive real root of (1.3). Namely, the best constant for inequality (3.2) is the real positive root of (1.3).

From (3.1) and above, we find that Theorem 1.1 holds.

Now we consider when we have equality in

$$(3.6) \quad \sum \frac{a}{b+c} \left(\tan^4 \frac{B}{2} + \tan^4 \frac{C}{2} \right) \geq \frac{1}{3} + \lambda_0 \left(1 - \frac{2r}{R} \right).$$

It is easy to see that equality in (3.6) holds when $\triangle ABC$ is an equilateral triangle. We consider another case: From the process of seeking $g_{min}(\delta)$ and Lemma 2.1, we can find the equality of inequality (3.6) holds when $\triangle ABC$ is an isosceles triangle with top-angle less than or equal to $\frac{\pi}{3}$ and $\delta = \delta_0$ or $\frac{2r}{R} = 2\delta_0 - \delta_0^2$, there is no harm in supposing $b = c = 1 (0 < a < 1)$, then

$$2\delta_0 - \delta_0^2 = \frac{2r}{R} = \frac{(a+b-c)(b+c-a)(c+a-b)}{abc} = a(2-a),$$

Thus $a = \delta_0$, namely, the equality of inequality (3.6) holds when $\triangle ABC$ is isosceles and the ratio of its side-lengths is $\delta_0 : 1 : 1$. \square

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4. The Proof Of Theorem 1.2

Proof. By Lemma 2.2 and Lemma 2.4,

$$(4.1) \quad \sum \frac{a}{b+c} \left(\tan^4 \frac{B}{2} + \tan^4 \frac{C}{2} \right) \\ \leq f(s_1) = \frac{\delta^8 + 5\delta^7 - 11\delta^6 - 123\delta^5 + 64\delta^4 + 1168\delta^3 - 2176\delta^2 + 512\delta + 512}{(\delta^4 - 5\delta^3 + 12\delta^2 - 40\delta + 64)(\delta - 4)^2}.$$

Now we prove

$$(4.2) \quad \frac{-2(\delta^8 + 5\delta^7 - 11\delta^6 - 123\delta^5 + 64\delta^4 + 1168\delta^3 - 2176\delta^2 + 512\delta + 512)}{(\delta^4 - 5\delta^3 + 12\delta^2 - 40\delta + 64)(\delta - 4)^2} \\ \leq \frac{1}{3} + \frac{8}{3} \left[\left(\frac{R}{2r} \right)^2 - 1 \right] = \frac{1}{3} + \frac{8}{3} \left[\left(\frac{1}{2\delta - \delta^2} \right)^2 - 1 \right].$$

Inequality (4.2) is equivalent to

$$(4.3) \quad \frac{(\delta - 1)X}{3\delta^2(\delta - 2)^2(\delta - 4)^2(\delta^4 - 5\delta^3 + 12\delta^2 - 40\delta + 64)} \geq 0,$$

where

$$(4.4) \quad X = 6\delta^{11} + 12\delta^{10} - 157\delta^9 - 392\delta^8 + 1812\delta^7 + 8112\delta^6 - 43416\delta^5 \\ + 70048\delta^4 - 46400\delta^3 + 12800\delta^2 + 1024\delta - 8192.$$

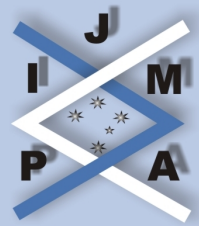
From $0 < \delta \leq 1$, it is easy to see that $t = \frac{1}{\delta} - 1 \geq 0$, hence, we can easily obtain the following two inequalities

$$(4.5) \quad \delta^4 - 5\delta^3 + 12\delta^2 - 40\delta + 64 = (1 - \delta)^4 + (1 - \delta)^3 + 3(1 - \delta)^2 + 27(1 - \delta) + 32 > 0$$

and

$$(4.6) \quad X = \delta^{11}(-8192t^{11} - 89088t^{10} - 427520t^9 - 1236800t^8 \\ - 2420832t^7 - 3346744t^6 - 3293632t^5 - 2280708t^4 \\ - 1080664t^3 - 332453t^2 - 59702t - 4743) < 0.$$

For $0 < \delta \leq 1$, together with (4.4) – (4.6), we can conclude that inequality (4.3) holds, so inequality (4.2) holds. Inequality (1.4) immediately follows from (4.1) and (4.2). \square



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References

- [1] O. BOTTEMA, R.Ž. DJORDEVIĆ, R.R. JANIĆ, D.S. MITRINOVIĆ AND P.M.VASIĆ, *Geometric Inequalities*, Wolters-Noordhoff Publishing, Groningen, The Netherlands, 1969.
- [2] W.-D. JIANG, A triangle inequality, Problem 4, (2008), *RGMIA Problem Corner*. [ONLINE <http://eureka.vu.edu.au/~rgmia/probcorner/2008/problem4-08.pdf>]
- [3] D.S. MITRINOVIĆ, J.E. PEČARIĆ AND V. VOLENEC, *Recent Advances in Geometric Inequalities*, Acad. Publ., Dordrecht, Boston, London, 1989.
- [4] MANH DUNG NGUYEN AND DUY KHANH NGUYEN, Problem 4, (2008), Solution No. 1, *RGMIA Problem Corner*. [ONLINE <http://eureka.vu.edu.au/~rgmia/probcorner/2008/problem4-08-sol1.pdf>].
- [5] S.-H. WU, A sharpened version of the fundamental triangle inequality, *Math. Inequal. Appl.*, **11**(3) (2008), 477–482.
- [6] S.-H. WU AND M. BENCZE, An equivalent form of the fundamental triangle inequality and its applications, *J. Inequal. Pure Appl. Math.*, **10**(1) (2009), Art. 16. [ONLINE <http://jipam.vu.edu.au/article.php?sid=1072>].
- [7] S.-H. WU AND Z.-H. ZHANG, A class of inequalities related to the angle bisectors and the sides of a triangle, *J. Inequal. Pure Appl. Math.*, **7**(3) (2006), Art. 108. [ONLINE <http://jipam.vu.edu.au/article.php?sid=698>].



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- [8] S.-H. WU AND Z.-H. ZHANG, Some strengthened results on Gerretsen's inequality, *RGMA Res. Rep. Coll.*, **6**(3) (2003), Art. 16. [ONLINE <http://www.staff.vu.edu.au/RGMIA/v6n3.asp>].
- [9] Y.-D. WU, On the open Question OQ.1285, *Octagon Mathematical Magazine*, **12**(2) (2004), 1041–1045.
- [10] L. YANG, J.-Z. ZHANG AND X.-R. HOU, *Nonlinear Algebraic Equation System and Automated Theorem Proving*, Shanghai Scientific and Technological Education Press (1996), 23–25. (in Chinese)

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