

Journal of Inequalities in Pure and Applied Mathematics

THE BEST CONSTANT FOR A GEOMETRIC INEQUALITY

YU-DONG WU

Xinchang Middle School
Xinchang, Zhejiang 312500
People's Republic of China.

EMail: zjxcwyd@tom.com

©2000 Victoria University
ISSN (electronic): 1443-5756
138-05



volume 6, issue 4, article 111,
2005.

*Received 29 April, 2005;
accepted 02 September, 2005.*

Communicated by: J. Sándor

Abstract

Contents



Home Page

Go Back

Close

Quit

Abstract

In this paper, we prove that the best constant for the geometric inequality $\frac{11\sqrt{3}}{5R+12r+k(2r-R)} \leq \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$ is a root of one polynomial by the method of mathematical analysis and linear algebra.

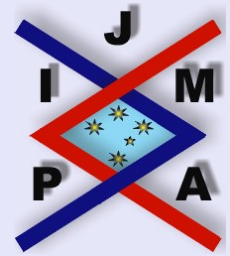
2000 Mathematics Subject Classification: Primary 52A40. Secondary 52C05.

Key words: Best Constant, Geometric Inequality, Euler's Inequality, Gerretsen's Inequality, Sylvester's Resultant.

The author would like to thank Professor Lu Yang and Sheng-Li Chen for their enthusiastic help.

Contents

1	Introduction and Main Results	3
2	Lemmas	5
3	Proof of Theorem 1.1	11
	References	



The Best Constant for a Geometric Inequality

Yu-dong Wu

Title Page

Contents



Go Back

Close

Quit

Page 2 of 17

1. Introduction and Main Results

In 1993, Shi-Chang Shi strengthened the familiar geometric inequality (in triangle)

$$(1.1) \quad \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \leq \frac{\sqrt{3}}{2r}$$

to

$$(1.2) \quad \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \leq \frac{1}{\sqrt{3}} \left(\frac{1}{r} + \frac{1}{R} \right)$$

in [1]. After several months, Ji Chen obtained the following beautiful and strong inequality chain in [2].

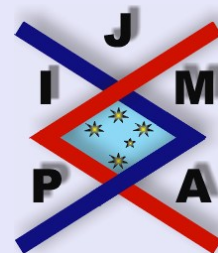
$$(1.3) \quad \frac{11\sqrt{3}}{5R + 12r} \leq \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \leq \frac{1}{\sqrt{3}} \left(\frac{5}{4R} + \frac{7}{8r} \right).$$

In the same year, Xi-Ling Huang posed the following interesting inequality problem in [3].

Problem 1. Determine the best constant k for which the inequality below holds

$$(1.4) \quad \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \leq \frac{1}{\sqrt{3}} \left[\frac{1}{R} + \frac{1}{r} + \frac{1}{k} \left(\frac{2}{R} - \frac{1}{r} \right) \right].$$

In 1996, Sheng-Li Chen solved Problem 1 completely in [4]. He obtained the following theorem.



The Best Constant for a Geometric Inequality

Yu-dong Wu

Title Page

Contents



Go Back

Close

Quit

Page 3 of 17

Theorem 1.1. *The best constant k for the inequality (1.2) is $2(1 + \sqrt[3]{2} + \sqrt[3]{4})$.*

In the same year, Xue-Zhi Yang [5] strengthened the inequality

$$(1.5) \quad \frac{11\sqrt{3}}{5R + 12r} \leq \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$$

to

$$(1.6) \quad \frac{243\sqrt{3}}{110R + 266r} \leq \frac{1}{a} + \frac{1}{b} + \frac{1}{c}.$$

In this paper we will determine the best constant for the inequality

$$(1.7) \quad \frac{11\sqrt{3}}{5R + 12r + k(2r - R)} \leq \frac{1}{a} + \frac{1}{b} + \frac{1}{c},$$

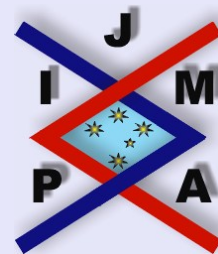
where $0 < k < 5$. We obtain the following theorem.

Theorem 1.2. *The maximum value of k for which the inequality (1.7) holds is the root on the open interval $(0, \frac{1}{15})$ of the following equation*

$$405k^5 + 6705k^4 + 129586k^3 + 1050976k^2 + 2795373k - 62181 = 0.$$

Its approximation is 0.02206078402.

In fact, let $k = \frac{5}{243} \approx 0.020576131687 < 0.02206078402$, we immediately find that the inequality (1.7) is just the inequality (1.6).



The Best Constant for a
Geometric Inequality

Yu-dong Wu

Title Page

Contents



Go Back

Close

Quit

Page 4 of 17

2. Lemmas

In order to prove Theorem 1.1, we require several lemmas. The second was obtained by Sheng-Li Chen in [6] (see also [7]).

Lemma 2.1. *If $0 < k < 5$, then the inequality*

$$(2.1) \quad \frac{11\sqrt{3}}{5R + 12r + k(2r - R)} \leq \frac{1}{\sqrt{3}} \left(\frac{5}{4R} + \frac{7}{8r} \right)$$

holds if and only if $0 < k \leq \frac{5}{12}$.

Proof. Since $0 < k < 5$, it is obvious that $5R + 12r + k(2r - R) > 0$. Therefore,

(2.1) is equivalent to

$$(2.2) \quad 7(5 - k)R^2 + (4k - 130)Rr + (20k + 120)r^2 \geq 0.$$

Setting $\frac{R}{2r} = x$, then with Euler's Inequality $R \geq 2r$, we have $x \geq 1$. Inequality (2.2) is equivalent to

$$28(5 - k)x^2 + 2(4k - 130)x + 20k + 120 \geq 0,$$

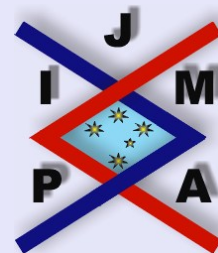
that is,

$$(2.3) \quad 4(x - 1)[(35 - 7k)x - 5k - 30] \geq 0.$$

Considering that $x \geq 1$, (2.3) holds if and only if $(35 - 7k)x - 5k - 30 \geq 0$ ($x \geq 1$). Namely, $k \leq \frac{5(7x-6)}{7x+5}$ or $k \leq \min \frac{5(7x-6)}{7x+5}$ ($x \geq 1$).

Define the function

$$f(x) = \frac{5(7x - 6)}{7x + 5} (x \geq 1).$$



The Best Constant for a
Geometric Inequality

Yu-dong Wu

Title Page

Contents



Go Back

Close

Quit

Page 5 of 17

Calculating the derivative for $f(x)$, we get

$$f'(x) = -\frac{35(7x-6)}{(7x+5)^2} + \frac{35}{7x+5} = \frac{385}{(7x+5)^2} > 0,$$

and thus the function $f(x)$ is strictly monotone increasing on the interval $[1, +\infty)$. Then $f(x) \geq f(1) = \frac{5}{12}$. That is $\min f(x) = \frac{5}{12}$ for $x \geq 1$. So $k \leq \frac{5}{12}$, combining $0 < k < 5$, we immediately obtain $0 < k \leq \frac{5}{12}$. Thus, Lemma 2.1 is proved. \square

Lemma 2.2. [6] *The homogeneous inequality $F(R, r, s) \geq (>)0$ in triangle which form is equivalent to $p \geq (>)f(R, r)$ holds if and only if it holds by setting $R = 2$, $r = 1 - x^2$, $p = \sqrt{(1-x)(3+x)^3}$, where $0 \leq x < 1$. And the form which is equivalent to $p \leq (<)f(R, r)$ holds if and only if it holds by setting the same substitution, where $-1 < x \leq 0$.*

Proof. It is well known that the following two inequalities

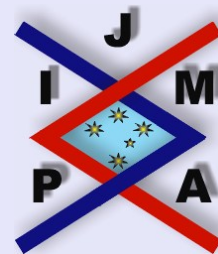
$$(2.4) \quad p^2 \geq 2R^2 + 10Rr - r^2 - 2(R-2r)\sqrt{r(R-2r)}$$

and

$$(2.5) \quad p^2 \leq 2R^2 + 10Rr - r^2 + 2(R-2r)\sqrt{r(R-2r)}$$

hold in any triangle ABC .

Now we prove the inequality (2.4) with equality holding if and only if $\triangle ABC$ is an isosceles triangle whose top-angle is greater than or equal to 60° , and the inequality (2.5) with equality holding if and only if $\triangle ABC$ is an isosceles



The Best Constant for a Geometric Inequality

Yu-dong Wu

Title Page

Contents



Go Back

Close

Quit

Page 6 of 17

triangle whose top-angle is less than or equal to 60° .

Let A be the top- angle of isosceles triangle ABC , and let

$$t = \sin \frac{A}{2} (= \cos B = \cos C) \in (0, 1),$$

then

$$\sin \frac{B}{2} = \sin \frac{C}{2} = \sqrt{\frac{1-t}{2}}.$$

With known identities

$$r = 4R \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}, \quad p = R(\sin A + \sin B + \sin C).$$

in triangle, we easily obtain

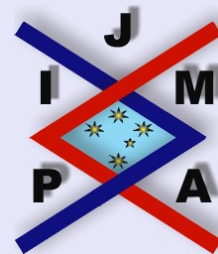
$$(2.6) \quad r = 2Rt(1-t), \quad p = 2R(1+t)\sqrt{1-t^2}.$$

We put the identities (2.6) into the inequality (2.4) and (2.5), with simple calculations, we can find the inequality (2.4) with equality holding if and only if $t \in [\frac{1}{2}, 1)$ or $A \geq 60^\circ$; the inequality (2.5) with equality holding if and only if $t \in (0, \frac{1}{2}]$ or $A \leq 60^\circ$.

Then we prove the following two propositions.

Proposition 2.3. *For every triangle ABC , there are isosceles triangle $A_1B_1C_1$ with top angle $A_1 \geq 60^\circ$ and isosceles triangle $A_2B_2C_2$ with top angle $A_2 \leq 60^\circ$ make*

$$R_1 = R_2 = R, \quad r_1 = r_2 = r; \quad p_1 \leq p \leq p_2,$$



The Best Constant for a Geometric Inequality

Yu-dong Wu

Title Page

Contents



Go Back

Close

Quit

Page 7 of 17

with $p = p_1$ holding if and only if $\triangle ABC$ is an isosceles triangle with top angle $A \geq 60^\circ$, $p = p_2$ holding if and only if $\triangle ABC$ is an isosceles triangle with top angle $A \leq 60^\circ$.

Proof. Denote $\odot O$ as the circumcircle of $\triangle ABC$, then there are inscribed isosceles triangles $A_1B_1C_1$ and $A_2B_2C_2$ of $\odot O$ which satisfy the next two identities:

$$\frac{A_1}{2} = \arcsin \frac{1}{2} \left(1 + \sqrt{1 - \frac{2r}{R}} \right),$$

$$\frac{A_2}{2} = \arcsin \frac{1}{2} \left(1 - \sqrt{1 - \frac{2r}{R}} \right).$$

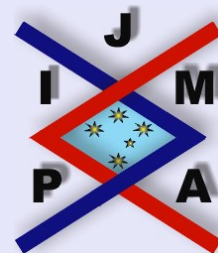
Then $A_1 \geq 60^\circ$, $A_2 \leq 60^\circ$ and

$$(2.7) \quad \sin \frac{A_1}{2} \left(1 - \sin \frac{A_1}{2} \right) = \frac{r}{2R},$$

$$(2.8) \quad \sin \frac{A_2}{2} \left(1 - \sin \frac{A_2}{2} \right) = \frac{r}{2R}.$$

For isosceles triangles $A_1B_1C_1$ where the top-angle is A_1 and $A_2B_2C_2$ where the top-angle is A_2 , we have

$$(2.9) \quad \sin \frac{A_1}{2} \left(1 - \sin \frac{A_1}{2} \right) = \frac{r_1}{2R_1},$$



The Best Constant for a Geometric Inequality

Yu-dong Wu

Title Page

Contents



Go Back

Close

Quit

Page 8 of 17

$$(2.10) \quad \sin \frac{A_2}{2} \left(1 - \sin \frac{A_2}{2} \right) = \frac{r_2}{2R_2}.$$

From (2.7) to (2.10), we get $\frac{r}{R} = \frac{r_1}{R_1} = \frac{r_2}{R_2}$, and it is easy to see that $R = R_1 = R_2$, so $r_1 = r_2 = r$. Denote $\varphi(R, r)$ to be the right of (2.4), then $p^2 \geq \varphi(R, r) = \varphi(R_1, r_1) = p_1^2$, so $p \geq p_1$. In the same manner, we can prove that $p \leq p_2$. \square

Proposition 2.4.

- (i) *If the inequality $p \geq (>)f_1(R, r)$ holds for any isosceles triangle whose top-angle is greater than or equal to 60° , then the inequality $p \geq (>)f_1(R, r)$ holds for any triangle.*
- (ii) *If the inequality $p \leq (<)f_1(R, r)$ holds for any isosceles triangle whose top-angle is less than or equal to 60° , then the inequality $p \leq (<)f_1(R, r)$ holds for any triangle.*

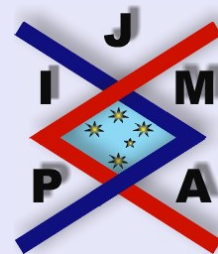
Proof. For any $\triangle A'B'C'$, with Proposition 2.3, we know there is an isosceles triangle $A_1B_1C_1$ which make

$$R_1 = R', \quad r_1 = r', \quad p_1 \leq p'.$$

Because the inequality $p \geq (>)f_1(R, r)$ holds for isosceles triangle $A_1B_1C_1$, we have

$$p' \geq p_1 \geq (>)f_1(R_1, r_1) = f_1(R', r').$$

Thus, the inequality $p \geq (>)f_1(R, r)$ holds for $\triangle A'B'C'$. In the same way we can prove (ii). \square



The Best Constant for a Geometric Inequality

Yu-dong Wu

Title Page

Contents



Go Back

Close

Quit

Page 9 of 17

From Proposition 2.4, the homogeneous inequality in triangle whose form is equivalent to $p \geq (>)f(R, r)$ holds if and only if it holds by setting $R = 2$, $r = 4t(1 - t)$, $p = 4(1 + t)\sqrt{1 - t^2}$. Taking $t = \frac{x+1}{2}$, we immediately get $r = 1 - x^2$, $p = \sqrt{(1 - x)(3 + x)^3}$, where $0 \leq x < 1$. For the homogeneous inequality in triangle whose form is equivalent to $p \leq (<)f(R, r)$, we only need to change the range of x . Namely, we change $0 \leq x < 1$ to be $-1 < x \leq 0$.

Thus, the proof of Lemma 2.2 is completed. (The proof was given by Sheng-Li Chen in [6].) \square

Lemma 2.5. [8] Denote

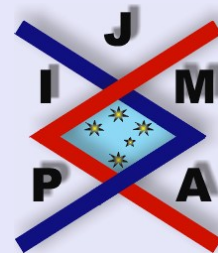
$$f(x) = a_0x^n + a_1x^{n-1} + \cdots + a_n,$$

$$g(x) = b_0x^m + b_1x^{m-1} + \cdots + b_m.$$

If $a_0 \neq 0$ or $b_0 \neq 0$, then the polynomials $f(x)$ and $g(x)$ have a common root if and only if

$$R(f, g) = \begin{vmatrix} a_0 & a_1 & a_2 & \cdots & a_n & 0 & \cdots & 0 \\ 0 & a_0 & a_1 & \cdots & a_{n-1} & a_n & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & a_0 & \cdots & \cdots & \cdots & a_n \\ b_0 & b_1 & b_2 & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & b_0 & b_1 & \cdots & \cdots & \cdots & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & b_0 & b_1 & \cdots & b_m \end{vmatrix} = 0,$$

where $R(f, g)$ is Sylvester's Resultant of $f(x)$ and $g(x)$.



The Best Constant for a Geometric Inequality

Yu-dong Wu

Title Page

Contents



Go Back

Close

Quit

Page 10 of 17

3. Proof of Theorem 1.1

Proof. With known identities $abc = 4Rrp$ and $ab + bc + ca = p^2 + 4Rr + r^2$ in triangle, we easily know the inequality (1.7) is equivalent to

$$(3.1) \quad \frac{11\sqrt{3}}{5R + 12r + k(2r - R)} \leq \frac{p^2 + 4Rr + r^2}{4Rrp}.$$

The inequality (3.1) is equivalent to the following inequality

$$(3.2) \quad [5R + 12r + k(2r - R)]p^2 - 44\sqrt{3}Rrp + [5R + 12r + k(2r - R)](4Rr + r^2) \geq 0.$$

(i) If

$$\Delta(R, r) = (44\sqrt{3}Rr)^2 - 4[5R + 12r + k(2r - R)]^2(4Rr + r^2) < 0,$$

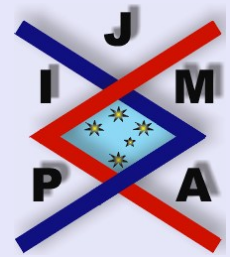
it is obvious that the inequality (3.2) holds.

(ii) If

$$\Delta(R, r) = (44\sqrt{3}Rr)^2 - 4[5R + 12r + k(2r - R)]^2(4Rr + r^2) \geq 0,$$

then the inequality (3.2) is equivalent to

$$(3.3) \quad p \geq \frac{44\sqrt{3}Rr + \sqrt{\Delta(R, r)}}{2[5R + 12r + k(2r - R)]}$$



The Best Constant for a Geometric Inequality

Yu-dong Wu

Title Page

Contents

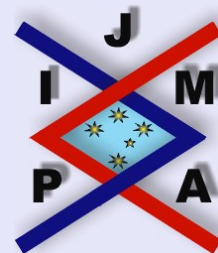


Go Back

Close

Quit

Page 11 of 17



Title Page

Contents



Go Back

Close

Quit

Page 12 of 17

or

$$(3.4) \quad p \leq \frac{44\sqrt{3}Rr - \sqrt{\Delta(R, r)}}{2[5R + 12r + k(2r - R)]}.$$

In fact, the inequality (3.4) does not hold. From (1.3) and (1.7), we have

$$(3.5) \quad \frac{11\sqrt{3}}{5R + 12r + k(2r - R)} \leq \frac{1}{\sqrt{3}} \left(\frac{5}{4R} + \frac{7}{8r} \right).$$

By Lemma 2.1, we know that $0 < k \leq \frac{5}{12}$. It is easy to see that the following inequalities hold

$$(3.6) \quad \frac{44\sqrt{3}Rr - \sqrt{\Delta(R, r)}}{2[5R + 12r + k(2r - R)]} \leq \frac{44\sqrt{3}Rr}{2[5R + 12r + k(2r - R)]} \\ \leq \frac{22\sqrt{3}Rr}{[5R + 12r + \frac{5}{12}(2r - R)]}.$$

Now we prove the next inequality

$$(3.7) \quad p \geq \frac{22\sqrt{3}Rr}{[5R + 12r + \frac{5}{12}(2r - R)]}.$$

The inequality (3.7) is equivalent to

$$(3.8) \quad p^2 \geq \frac{1452R^2r^2}{[5R + 12r + \frac{5}{12}(2r - R)]^2}.$$

With Gerretsen's Inequality $p^2 \geq 16Rr - 5r^2$, in order to prove the inequality (3.8), we only need to prove the following inequality.

$$(3.9) \quad 16Rr - 5r^2 \geq \frac{1452R^2r^2}{\left[5R + 12r + \frac{5}{12}(2r - R)\right]^2}.$$

The inequality (3.9) is equivalent to

$$(3.10) \quad r(400R^3 + 387R^2 + 2436Rr - 980r^3) \geq 0.$$

With Euler's inequality $R \geq 2r$, we easily see that the inequality (3.10) holds. So, the inequality (3.7) holds. Then the inequality (3.4) does not hold. Therefore, the inequality (3.2) is equivalent to the inequality (3.3). From Lemma 2.2, the inequality (3.2) holds if and only if the following inequality holds.

$$(3.11) \quad 8(1-x)(3+x) \left[(2x+3)(11-6x^2-kx^2) - 11\sqrt{3}(x+1)\sqrt{(1-x)(3+x)} \right] \geq 0 \quad (0 \leq x < 1).$$

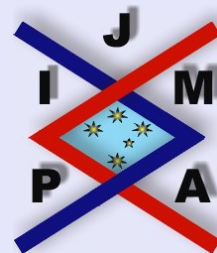
The inequality (3.11) holds when $x = 0$. When $0 < x < 1$, the inequality (3.11) is equivalent to

$$(3.12) \quad k \leq \frac{(2x+3)(11-6x^2) - 11\sqrt{3}(x+1)\sqrt{(1-x)(3+x)}}{x^2(2x+3)}.$$

Define the function

$$(3.13) \quad g(x) = \frac{(2x+3)(11-6x^2) - 11\sqrt{3}(x+1)\sqrt{(1-x)(3+x)}}{x^2(2x+3)},$$

$$(0 < x < 1).$$



The Best Constant for a Geometric Inequality

Yu-dong Wu

Title Page

Contents



Go Back

Close

Quit

Page 13 of 17

Calculating the derivative for $g(x)$, we get

$$(3.14) \quad g'(x) = \frac{-22 \left[\sqrt{3}(x^4 + 5x^3 + 2x^2 - 9x - 9) + (2x + 3)^2 \sqrt{(1-x)(3+x)} \right]}{x^3(2x + 3)^2 \sqrt{(1-x)(3+x)}}.$$

Let $g'(x) = 0$, we get

$$(3.15) \quad 3x^5 + 30x^4 + 103x^3 + 134x^2 + 48x - 18 = 0, \quad (0 < x < 1).$$

It is easy to see that the equation (3.15) has the only one positive root on the open interval $(0, 1)$. Denote x_0 to be the root of the equation (3.15). Then

$$g(x)_{\min} = g(x_0) = \frac{(2x_0 + 3)(11 - 6x_0^2) - 11\sqrt{3}(x_0 + 1)\sqrt{(1-x_0)(3+x_0)}}{x_0^2(2x_0 + 3)}.$$

Therefore, the maximum of k is $g(x_0)$. Now we prove $g(x_0)$ is the root of the equation

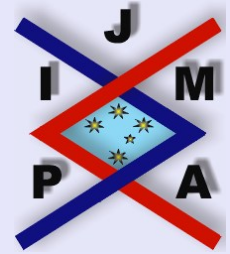
$$405k^5 + 6705k^4 + 129586k^3 + 1050976k^2 + 2795373k - 62181 = 0.$$

It is easy to find that $g(x_0)$ is a root of the following equation.

$$x_0^2(2x_0+3)^2t^2 - 2(2x_0+3)^2(11-6x_0^2)t + 144x_0^4 + 432x_0^3 + 159x_0^2 - 132x_0 + 22 = 0.$$

We know that

$$3x_0^5 + 30x_0^4 + 103x_0^3 + 134x_0^2 + 48x_0 - 18 = 0.$$



The Best Constant for a Geometric Inequality

Yu-dong Wu

Title Page

Contents



Go Back

Close

Quit

Page 14 of 17

Considering the simultaneous equations

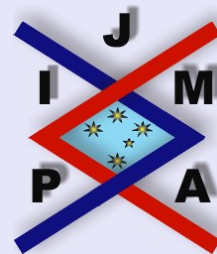
$$(3.16) \quad \begin{cases} x_0^2(2x_0 + 3)^2 t^2 - 2(2x_0 + 3)^2(11 - 6x_0^2)t + 144x_0^4 \\ \quad + 432x_0^3 + 159x_0^2 - 132x_0 + 22 = 0 \\ 3x_0^5 + 30x_0^4 + 103x_0^3 + 134x_0^2 + 48x_0 - 18 = 0 \end{cases}$$

The simultaneous equations (3.16) can be changed to the simultaneous equations as follows.

$$(3.17) \quad \begin{cases} 4(t + 6)^2 x_0^4 + 12(t + 6)^2 x_0^3 + (9t^2 + 20t + 159)x_0^2 \\ \quad - 132(2t + 1)x_0 - 198t + 22 = 0 \\ 3x_0^5 + 30x_0^4 + 103x_0^3 + 134x_0^2 + 48x_0 - 18 = 0 \end{cases}$$

Then,

$$R_{x_0}(f, g) = \begin{vmatrix} 4(t+6)^2 & 12(t+6)^2 & \cdots & 22-198t & 0 & \cdots & 0 \\ 0 & 4(t+6)^2 & \cdots & \cdots & 22-198t & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & 4(t+6)^2 & \cdots & \cdots & 22-198t \\ 3 & 30 & \cdots & -18 & 0 & 0 & 0 \\ 0 & 3 & 30 & \cdots & -18 & 0 & 0 \\ 0 & 0 & 3 & 30 & \cdots & -18 & 0 \\ 0 & 0 & 0 & 3 & 30 & \cdots & -18 \end{vmatrix}$$



The Best Constant for a Geometric Inequality

Yu-dong Wu

Title Page

Contents



Go Back

Close

Quit

Page 15 of 17

$$= 100(405t^5 + 6705t^4 + 129586t^3 + 1050976t^2 + 2795373t - 62181) \\ \times (405t^5 - 178425t^4 - 1656374t^3 \\ - 13317290t^2 - 100675599t - 330639021).$$

The solution of the equation $R_{x_0}(f, g) = 0$ is the union of the solution of the equation

$$(3.18) \quad 405t^5 + 6705t^4 + 129586t^3 + 1050976t^2 + 2795373t - 62181 = 0,$$

and the equation

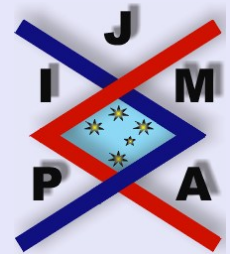
$$(3.19) \quad 405t^5 - 178425t^4 - 1656374t^3 \\ - 13317290t^2 - 100675599t - 330639021 = 0.$$

With differential calculus, it is easy to see that the equation (3.19) has no root on the interval $[0, 1]$. We can get $g(x_0) < 1$, with Lemma 2.5, we can conclude that $g(x_0)$ is the real root of the equation (3.18). Define the function

$$(3.20) \quad f(t) = 405t^5 + 6705t^4 + 129586t^3 + 1050976t^2 + 2795373t - 62181.$$

Then $f(\frac{1}{15}) = \frac{2174963624}{16875} > 0$. Therefore, the real root of the equation (3.18) is on the interval $(0, \frac{1}{15})$.

Thus, the proof of Theorem 1.2 is completed. \square



The Best Constant for a
Geometric Inequality

Yu-dong Wu

Title Page

Contents



Go Back

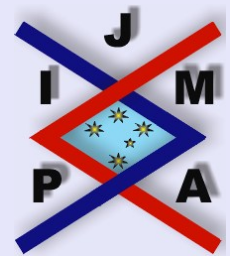
Close

Quit

Page 16 of 17

References

- [1] SHI-CHANG SHI, Strengthening on a geometric inequality, *Fujian High-School Mathematics* (in Chinese), **7**(4) (1993).
- [2] JI CHEN, Discussion about a geometric inequality (First), *Fujian High-School Mathematics* (in Chinese), **10-11**(6) (1993).
- [3] XI-LING HUANG, Discussion about a geometric inequality (Third), *Fujian High-School Mathematics* (in Chinese), **11**(6) (1993).
- [4] SHENG-LI CHEN, Discussing about a geometric inequality again, *Geometric Inequalities in China* (in Chinese), Jiangsu Educational Press, No. 46-50, 1996.
- [5] XUE-ZHI YANG, Strengthening on a geometric inequality (First), *Forward Position of Elementary Mathematics* (in Chinese), Jiangsu Educational Press, No. 236-242, 1996.
- [6] SHENG-LI CHEN, A new method to prove one kind of inequalities—equate substitution method (in Chinese), *Fujian High-School Mathematics*, No. 20-23, 1993(3).
- [7] SHENG-LI CHEN, The simplified method to prove inequalities in triangle (in Chinese), *Research in Inequalities*, Xizang People's Press, No. 3-8, 2000.
- [8] LU YANG, JING-ZHONG ZHANG AND XIAO-RONG HOU, *Nonlinear Algebraic Equation System and Automated Theorem Proving* (in Chinese), Shanghai Scientific and Technological Education Press, No. 23-25, 1996.



The Best Constant for a
Geometric Inequality

Yu-dong Wu

Title Page

Contents



Go Back

Close

Quit

Page 17 of 17