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INCLUSION AND NEIGHBORHOOD PROPERTIES OF CERTAIN SUBCLASSES OF ANALYTIC AND MULTIVALENT FUNCTIONS OF COMPLEX ORDER

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Abstract

In the present paper, the authors prove several inclusion relations associated with the (n, δ)-neighborhoods of certain subclasses of p-valently analytic functions of complex order, which are introduced here by means of a family of extended multiplier transformations. Special cases of some of these inclusion relations are shown to yield known results.

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Contents

1. Introduction, Definitions and Preliminaries

Let $\mathcal{A}_p(n)$ denote the class of functions $f(z)$ normalized by

$$f(z) = z^p - \sum_{\tau=n+p}^{\infty} a_\tau z^\tau \tag{1.1}$$

$$(a_\tau \geqq 0;\ n, p \in \mathbb{N} := \{1, 2, 3, \dots\}),$$

which are *analytic* and *p-valent* in the open unit disk

$$\mathbb{U} := \{z : z \in \mathbb{C} \quad \text{and} \quad |z| < 1\}.$$

Analogous to the multiplier transformation on $\mathcal{A}$, the operator $I_p(r, \mu)$, given on $\mathcal{A}_p(1)$ by

$$I_p(r, \mu)f(z) := z^p - \sum_{\tau=p+1}^{\infty} \left(\frac{\tau + \mu}{p + \mu}\right)^r a_\tau z^\tau$$

$$(\mu \geqq 0;\ r \in \mathbb{Z};\ f \in \mathcal{A}_p(1)),$$

was studied by Kumar *et al.* [6]. It is easily verified that

$$(p + \mu)I_p(r + 1, \mu)f(z) = z[I_p(r, \mu)f(z)]' + \mu I_p(r, \mu)f(z).$$

The operator $I_p(r, \mu)$ is closely related to the Sălăgean derivative operator [11]. The operator

$$I_\mu^r := I_1(r, \mu)$$

was studied by Cho and Srivastava [4] and Cho and Kim [3]. Moreover, the operator

$$I_r := I_1(r,1)$$

was studied earlier by Uraleggadi and Somanatha [13].

Here, in our present investigation, we define the operator $I_p(r,\mu)$ on $\mathcal{A}_p(n)$ by

$$I_p(r,\mu)f(z) := z^p - \sum_{\tau=n+p}^{\infty}\left(\frac{\tau+\mu}{p+\mu}\right)^r a_\tau z^\tau \tag{1.2}$$

$$(\mu \geqq 0;\; p \in \mathbb{N};\; r \in \mathbb{Z}).$$

By using the operator $I_p(r,\mu)f(z)$ given by (1.2), we introduce a subclass $\mathcal{S}^p_{n,m}(\mu,r,\lambda,b)$ of the p-valently analytic function class $\mathcal{A}_p(n)$, which consists of functions $f(z)$ satisfying the following inequality:

$$\left|\frac{1}{b}\left(\frac{z[I_p(r,\mu)f(z)]^{(m+1)}+\lambda z^2[I_p(r,\mu)f(z)]^{(m+2)}}{\lambda z[I_p(r,\mu)f(z)]^{(m+1)}+(1-\lambda)[I_p(r,\mu)f(z)]^{(m)}}-(p-m)\right)\right|<1 \tag{1.3}$$

$$(z\in\mathbb{U};\; p\in\mathbb{N};\; m\in\mathbb{N}_0;\; r\in\mathbb{Z};\; \mu\geqq 0;\; \lambda\geqq 0;\; p>\max(m,-\mu);\; b\in\mathbb{C}\setminus\{0\}).$$

Next, following the earlier investigations by Goodman [5], Ruscheweyh [10] and Altintas *et al.* [2] (see also [1], [7] and [12]), we define the (n,δ)-neighborhood of a function $f(z)\in\mathcal{A}_p(n)$ by (see, for details, [2, p. 1668])

$$N_{n,\delta}(f) := \left\{g\in\mathcal{A}_p(n) : g(z)=z^p-\sum_{\tau=n+p}^{\infty} b_\tau z^\tau \text{ and } \sum_{\tau=n+p}^{\infty}\tau|a_\tau-b_\tau|\leqq\delta\right\}. \tag{1.4}$$

It follows from (1.4) that, if

$$h(z) = z^p \quad (p \in \mathbb{N}), \tag{1.5}$$

then

$$N_{n,\delta}(h) := \left\{ g \in \mathcal{A}_p(n) : g(z) = z^p - \sum_{\tau=n+p}^{\infty} b_\tau z^\tau \text{ and } \sum_{\tau=n+p}^{\infty} \tau |b_\tau| \leqq \delta \right\}. \tag{1.6}$$

Finally, we denote by $\mathcal{R}^p_{n,m}(\mu, r, \lambda, b)$ the subclass of $\mathcal{A}_p(n)$ consisting of functions $f(z)$ which satisfy the inequality (1.7) below:

$$\left| \frac{1}{b} \{ [1 - \lambda(p - m - 1)][I_p(r, \mu) f(z)]^{(m+1)} + \lambda z [I_p(r, \mu) f(z)]^{(m+2)} - (p - m) \} \right| < p - m \tag{1.7}$$

$\left(z \in \mathbb{U};\ p \in \mathbb{N};\ m \in \mathbb{N}_0;\ r \in \mathbb{Z};\ \mu \geqq 0;\ \lambda \geqq 0;\ p > \max(m, -\mu);\ b \in \mathbb{C}\backslash\{0\}\right)$.

The object of the present paper is to investigate the various properties and characteristics of analytic p-valent functions belonging to the subclasses

$$\mathcal{S}^p_{n,m}(\mu, r, \lambda, b) \quad \text{and} \quad \mathcal{R}^p_{n,m}(\mu, r, \lambda, b),$$

which we have defined here. Apart from deriving a set of coefficient bounds for each of these function classes, we establish several inclusion relationships involving the (n, δ)-neighborhoods of analytic p-valent functions (with negative and missing coefficients) belonging to these subclasses.

Our definitions of the function classes

$$\mathcal{S}^p_{n,m}(\mu, r, \lambda, b) \quad \text{and} \quad \mathcal{R}^p_{n,m}(\mu, r, \lambda, b)$$

are motivated essentially by the earlier investigations of Orhan and Kamali [8], and of Raina and Srivastava [9], in each of which further details and closely-related subclasses can be found. In particular, in our definition of the function classes

$$\mathcal{S}^p_{n,m}(\mu, r, \lambda, b) \quad \text{and} \quad \mathcal{R}^p_{n,m}(\mu, r, \lambda, b)$$

involving the inequalities (1.3) and (1.7), we have relaxed the parametric constraint

$$0 \leqq \lambda \leqq 1,$$

which was imposed earlier by Orhan and Kamali [8, p. 57, Equations (1.10) and (1.11)] (see also Remark 3 below).

2. A Set of Coefficient Bounds

In this section, we prove the following results which yield the coefficient inequalities for functions in the subclasses

$$\mathcal{S}^p_{n,m}(\mu, r, \lambda, b) \quad \text{and} \quad \mathcal{R}^p_{n,m}(\mu, r, \lambda, b).$$

Theorem 1. *Let* $f(z) \in \mathcal{A}_p(n)$ *be given by* (1.1)*. Then* $f(z) \in \mathcal{S}^p_{n,m}(\mu, r, \lambda, b)$ *if and only if*

$$\sum_{\tau=n+p}^{\infty} \left(\frac{\tau+\mu}{p+\mu}\right)^r \binom{\tau}{m} [1+\lambda(\tau-m-1)](\tau-p+|b|)a_\tau \leqq |b| \left\{ \binom{p}{m} [1+\lambda(p-m-1)] \right\}, \tag{2.1}$$

where

$$\binom{\tau}{m} = \frac{\tau(\tau-1)\cdots(\tau-m+1)}{m!}.$$

Proof. Let a function $f(z)$ of the form (1.1) belong to the class $\mathcal{S}^p_{n,m}(\mu, r, \lambda, b)$. Then, in view of (1.2) and (1.3), we have the following inequality:

$$\Re\left(\frac{-\sum_{\tau=n+p}^{\infty} \left(\frac{\tau+\mu}{p+\mu}\right)^r \binom{\tau}{m} (\tau-p)[1+\lambda(\tau-m-1)]a_\tau z^{\tau-m}}{\binom{p}{m}[1+\lambda(p-m-1)]z^{p-m} - \sum_{\tau=n+p}^{\infty} \left(\frac{\tau+\mu}{p+\mu}\right)^r \binom{\tau}{m}[1+\lambda(\tau-m-1)]a_\tau z^{\tau-m}} \right) > -|b| \qquad (z \in \mathbb{U}). \tag{2.2}$$

Putting $z = r_1$ $(0 \leqq r_1 < 1)$ in (2.2), we observe that the expression in the denominator on the left-hand side of (2.2) is positive for $r_1 = 0$ and also for all r_1 $(0 < r_1 < 1)$. Thus, by letting $r_1 \to 1-$ through real values, (2.2) leads us to the desired assertion (2.1) of Theorem 1.

Conversely, by applying (2.1) and setting $|z| = 1$, we find by using (1.2) that

$$\left| \frac{z[I_p(r,\mu)f(z)]^{(m+1)} + \lambda z^2[I_p(r,\mu)f(z)]^{(m+2)}}{\lambda z[I_p(r,\mu)f(z)]^{(m+1)} + (1-\lambda)[I_p(r,\mu)f(z)]^{(m)}} - (p-m) \right|$$
$$= \left| \frac{\sum_{\tau=n+p}^{\infty} \left(\frac{\tau+\mu}{p+\mu}\right)^r \binom{\tau}{m}[1+\lambda(\tau-m-1)](\tau-p)a_\tau}{\binom{p}{m}[1+\lambda(p-m-1)] - \sum_{\tau=n+p}^{\infty} \left(\frac{\tau+\mu}{p+\mu}\right)^r \binom{\tau}{m}[1+\lambda(\tau-m-1)]a_\tau} \right|$$
$$\leqq \frac{|b| \left[\binom{p}{m}[1+\lambda(p-m-1)] - \sum_{\tau=n+p}^{\infty} \left(\frac{\tau+\mu}{p+\mu}\right)^r \binom{\tau}{m}[1+\lambda(\tau-m-1)]a_\tau \right]}{\binom{p}{m}[1+\lambda(p-m-1)] - \sum_{\tau=n+p}^{\infty} \left(\frac{\tau+\mu}{p+\mu}\right)^r \binom{\tau}{m}[1+\lambda(\tau-m-1)]a_\tau}$$
$$= |b|.$$

Hence, by the maximum modulus principle, we infer that $f(z) \in \mathcal{S}^p_{n,m}(\mu, r, \lambda, b)$, which completes the proof of Theorem 1. □

Remark 1. In the special case when

$$m = 0, \quad p = 1, \quad b = \beta\gamma \quad (0 < \beta \leqq 1; \quad \gamma \in \mathbb{C} \setminus \{0\}),$$
$$r = \Omega \quad (\Omega \in \mathbb{N}_0 := \mathbb{N} \cup \{0\}), \quad \tau = k+1, \quad \text{and} \quad \mu = 0, \tag{2.3}$$

Theorem 1 corresponds to a result given earlier by Orhan and Kamali [8, p. 57, Lemma 1].

By using the same arguments as in the proof of Theorem 1, we can establish Theorem 2 below.

Theorem 2. *Let* $f(z) \in \mathcal{A}_p(n)$ *be given by* (1.1). *Then* $f(z) \in \mathcal{R}^p_{n,m}(\mu, r, \lambda, b)$ *if and only if*

$$\sum_{\tau=n+p}^{\infty} \left(\frac{\tau+\mu}{p+\mu}\right)^r \binom{\tau}{m} (\tau - m)[1 + \lambda(\tau - p)]a_\tau \leqq (p-m)\left[\frac{|b|-1}{m!} + \binom{p}{m}\right]. \tag{2.4}$$

Remark 2. Making use of the same parametric substitutions as mentioned above in (2.3), Theorem 2 yields another known result due to Orhan and Kamali [8, p. 58, Lemma 2].

3. Inclusion Relationships Involving the (n, δ)-Neighborhoods

In this section, we establish several inclusion relationships for the function classes

$$\mathcal{S}^p_{n,m}(\mu, r, \lambda, b) \quad \text{and} \quad \mathcal{R}^p_{n,m}(\mu, r, \lambda, b)$$

involving the (n, δ)-neighborhood defined by (1.6).

Theorem 3. *If*

$$\delta := \frac{|b|(n+p)\binom{p}{m}[1+\lambda(p-m-1)]}{(n+|b|)\left(\frac{n+p+\mu}{p+\mu}\right)^r \binom{n+p}{m}[1+\lambda(n+p-m-1)]} \quad (p > |b|), \tag{3.1}$$

then

$$\mathcal{S}^p_{n,m}(\mu, r, \lambda, b) \subset N_{n,\delta}(h). \tag{3.2}$$

Proof. Let $f(z) \in \mathcal{S}^p_{n,m}(\mu, r, \lambda, b)$. Then, in view of the assertion (2.1) of Theorem 1, we have

$$(n+|b|)\left(\frac{n+p+\mu}{p+\mu}\right)^r \binom{n+p}{m}[1+\lambda(n+p-m-1)] \sum_{\tau=n+p}^{\infty} a_\tau$$
$$\leqq |b|\binom{p}{m}[1+\lambda(p-m-1)],$$

which yields

$$\sum_{\tau=n+p}^{\infty} a_\tau \leqq \frac{|b|\binom{p}{m}[1+\lambda(p-m-1)]}{(n+|b|)\left(\frac{n+p+\mu}{p+\mu}\right)^r \binom{n+p}{m}[1+\lambda(n+p-m-1)]}. \tag{3.3}$$

Applying the assertion (2.1) of Theorem 1 again, in conjunction with (3.3), we obtain

$$\begin{aligned}
&\left(\frac{n+p+\mu}{p+\mu}\right)^r \binom{n+p}{m}[1+\lambda(n+p-m-1)] \sum_{\tau=n+p}^{\infty} \tau a_\tau \\
&\leqq |b|\binom{p}{m}[1+\lambda(p-m-1)] + (p-|b|)\left(\frac{n+p+\mu}{p+\mu}\right)^r \\
&\qquad \cdot \binom{n+p}{m}[1+\lambda(n+p-m-1)] \sum_{\tau=n+p}^{\infty} a_\tau \\
&\leqq |b|\binom{p}{m}[1+\lambda(p-m-1)] + (p-|b|)\left(\frac{n+p+\mu}{p+\mu}\right)^r \\
&\qquad \cdot \binom{n+p}{m}[1+\lambda(n+p-m-1)] \\
&\qquad \cdot \frac{|b|\binom{p}{m}[1+\lambda(p-m-1)]}{(n+|b|)\left(\frac{n+p+\mu}{p+\mu}\right)^r \binom{n+p}{m}[1+\lambda(n+p-m-1)]} \\
&= |b|\binom{p}{m}[1+\lambda(p-m-1)]\left(\frac{n+p}{n+|b|}\right).
\end{aligned}$$

Hence

$$\sum_{\tau=n+p} \tau a_\tau \leqq \frac{|b|(n+p)\binom{p}{m}[1+\lambda(p-m-1)]}{(n+|b|)\left(\frac{n+p+\mu}{p+\mu}\right)^r \binom{n+p}{m}[1+\lambda(n+p-m-1)]} =: \delta \quad (p > |b|), \tag{3.4}$$

which, by virtue of (1.6), establishes the inclusion relation (3.2) of Theorem 3. □

Analogously, by applying the assertion (2.4) of Theorem 2 instead of the assertion (2.1) of Theorem 1 to functions in the class $\mathcal{R}^p_{n,m}(\mu, r, \lambda, b)$, we can prove the following inclusion relationship.

Theorem 4. *If*

$$\delta = \frac{(p-m)\left[\frac{|b|-1}{m!} + \binom{p}{m}\right]}{\left(\frac{n+p+\mu}{p+\mu}\right)^r \binom{n+p-1}{m}(1+\lambda n)} \qquad \left(\lambda > \frac{1}{p}\right), \tag{3.5}$$

then

$$\mathcal{R}^p_{n,m}(\mu, r, \lambda, b) \subset N_{n,\delta}(h). \tag{3.6}$$

Remark 3. Applying the parametric substitutions listed in (2.3), Theorem 3 and Theorem 4 would yield the known results due to Orhan and Kamali [8, p. 58, Theorem 1; p. 59, Theorem 2]. Incidentally, just as we indicated in Section 1 above, the condition $\lambda > 1$ is needed in the proof of one of these known results [8, p. 59, Theorem 2]. This implies that the constraint $0 \leqq \lambda \leqq 1$ in [8, p. 57, Equations (1.10) and (1.11)] should be replaced by the less stringent constraint $\lambda \geqq 0$.

4. Further Neighborhood Properties

In this last section, we determine the neighborhood properties for each of the following (slightly modified) function classes:

$$\mathcal{S}^{p,\alpha}_{n,m}(\mu, r, \lambda, b) \quad \text{and} \quad \mathcal{R}^{p,\alpha}_{n,m}(\mu, r, \lambda, b).$$

Here the class $\mathcal{S}^{p,\alpha}_{n,m}(\mu, r, \lambda, b)$ consists of functions $f(z) \in \mathcal{A}_p(n)$ for which there exists another function $g(z) \in \mathcal{S}^{p}_{n,m}(\mu, r, \lambda, b)$ such that

$$\left| \frac{f(z)}{g(z)} - 1 \right| < p - \alpha \qquad (z \in \mathbb{U};\ 0 \leqq \alpha < p) \tag{4.1}$$

Analogously, the class $\mathcal{R}^{p,\alpha}_{n,m}(\mu, r, \lambda, b)$ consists of functions $f(z) \in \mathcal{A}_p(n)$ for which there exists another function $g(z) \in \mathcal{R}^{p}_{n,m}(\mu, r, \lambda, b)$ satisfying the inequality (4.1).

Theorem 5. *Let $g(z) \in \mathcal{S}^{p}_{n,m}(\mu, r, \lambda, b)$. Suppose also that*

$$\alpha = p - \frac{\delta}{n+p} \tag{4.2}$$

$$\cdot \left[\frac{(n+|b|)\left(\frac{n+p+\mu}{p+\mu}\right)^r \binom{n+p}{m}[1+\lambda(n+p-m-1)]}{(n+|b|)\left(\frac{n+p+\mu}{p+\mu}\right)^r \binom{n+p}{m}[1+\lambda(n+p-m-1)] - |b|\binom{p}{m}[1+\lambda(p-m-1)]} \right].$$

Then

$$N_{n,\delta}(g) \subset \mathcal{S}^{p,\alpha}_{n,m}(\mu, r, \lambda, b). \tag{4.3}$$

Proof. Suppose that $f(z) \in N_{n,\delta}(g)$. We then find from (1.4) that

$$\sum_{\tau=n+p}^{\infty} \tau |a_\tau - b_\tau| \leqq \delta, \tag{4.4}$$

which readily implies the following coefficient inequality:

$$\sum_{\tau=n+p}^{\infty} |a_\tau - b_\tau| \leqq \frac{\delta}{n+p} \quad (n \in \mathbb{N}). \tag{4.5}$$

Next, since $g \in \mathcal{S}^p_{n,m}(\mu, r, \lambda, b)$, we have

$$\sum_{\tau=n+p}^{\infty} b_\tau \leqq \frac{|b|\binom{p}{m}[1+\lambda(p-m-1)]}{(n+|b|)\left(\frac{n+p+\mu}{p+\mu}\right)^r \binom{n+p}{m}[1+\lambda(n+p-m-1)]}, \tag{4.6}$$

so that

$$\begin{aligned}
\left|\frac{f(z)}{g(z)} - 1\right| &< \frac{\sum_{\tau=n+p}^{\infty} |a_\tau - b_\tau|}{1 - \sum_{\tau=n+p}^{\infty} b_\tau} \\
&\leqq \frac{\delta}{n+p}\left[1 - \frac{|b|\binom{p}{m}[1+\lambda(p-m-1)]}{(n+|b|)\left(\frac{n+p+\mu}{p+\mu}\right)^r \binom{n+p}{m}[1+\lambda(n+p-m-1)]}\right]^{-1} \\
&= \frac{\delta}{n+p}\left[\frac{(n+|b|)\left(\frac{n+p+\mu}{p+\mu}\right)^r \binom{n+p}{m}[1+\lambda(n+p-m-1)]}{(n+|b|)\left(\frac{n+p+\mu}{p+\mu}\right)^r \binom{n+p}{m}[1+\lambda(n+p-m-1)] - |b|\binom{p}{m}[1+\lambda(p-m-1)]}\right] \\
&= p - \alpha,
\end{aligned}$$

provided that α is given precisely by (4.2). Thus, by definition, $f \in \mathcal{S}_{n,m}^{p,\alpha}(\mu, r, \lambda, b)$ for α given by (4.2). This evidently completes the proof of Theorem 5. □

The proof of Theorem 6 below is much similar to that of Theorem 5; hence the proof of Theorem 6 is being omitted.

Theorem 6. *Let* $g(z) \in \mathcal{R}_{n,m}^{p,\alpha}(\mu, r, \lambda, b)$. *Suppose also that*

$$\alpha = p - \frac{\delta}{n+p} \times \left[\frac{\left(\frac{n+p+\mu}{p+\mu}\right)^r \binom{n+p}{m}(n+p-m)(1+\lambda n)}{\left(\frac{n+p+\mu}{p+\mu}\right)^r \binom{n+p}{m}(n+p-m)(1+\lambda n) - (p-m)\left[\frac{|b|-1}{m!} + \binom{p}{m}\right]} \right]. \tag{4.7}$$

Then

$$N_{n,\delta}(g) \subset \mathcal{R}_{n,m}^{p,\alpha}(\mu, r, \lambda, b). \tag{4.8}$$

Remark 4. Applying the parametric substitutions listed in (2.3), Theorem 5 and Theorem 6 would yield the known results due to Orhan and Kamali [8, p. 60, Theorem 3; p. 61, Theorem 4].

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