ON THE EXISTENCE OF SOLUTIONS TO A CLASS OF p-LAPLACE ELLIPTIC EQUATIONS

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Abstract: We study the equation $-\Delta_p u + |x|^a |u|^{p-2} u = |x|^b |u|^{q-2} u$ with Dirichlet boundary condition on $B(0, R)$ or on $\mathbb{R}^N$. We prove the existence of the radial solution and nonradial solutions of this equation.

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1. Introduction and Main Result

Equations of the form

$$(1.1) \quad \begin{cases} -\Delta_t u + g(x)|u|^{s-2}u = f(x,u) & \text{in } \Omega \\ u = 0, & \text{on } \partial\Omega \end{cases}$$

have attracted much attention. Many papers deal with the problem (1.1) in the case of $t = 2$, $\Omega = \mathbb{R}^N$, $s = 2$, g large at infinity and f superlinear, subcritical and bounded in x, see e.g. [1], [2] and [4]. The problem (1.1) with $t = 2$, $\Omega = B(0,1)$, $g(x) = 0$ and $f(x,u) = |x|^b u^{l-1}$ was studied in [9]; in particular, it was proved that under some conditions the ground states are not radial symmetric. The case $t = 2$, $\Omega = B(0,1)$ or on $\mathbb{R}^N$, $g(x) = 1$ and $f(x,u) = |x|^b|u|^{l-2}u$ was studied in [7]. The problem (1.1) with $t = s$, $\Omega = \mathbb{R}^N$, $g(x) = V(|x|)$ and $f(x,u) = Q(|x|)|u|^{l-2}u$ was studied by J. Su., Z.-Q. Wang and M. Willem ([11], [12]). They proved embedding results for functions in the weighted $W^{1,p}(\mathbb{R}^N)$ space of radial symmetry. The results were then used to obtain ground state and bound state solutions of equations with unbounded or decaying radial potentials.

In this paper, we consider the nonlinear elliptic problem

$$(1.2) \quad \begin{cases} -\Delta_p u + |x|^a|u|^{p-2}u = |x|^b|u|^{q-2}u & \text{in } \Omega \\ u > 0, \ u \in W^{1,p}(\Omega), \\ u = 0, & \text{on } \partial\Omega \end{cases}$$

and prove the existence of the radial and the nonradial solutions of the problem (1.2). Here, $\Delta_p u = div(|\nabla u|^{p-2}\nabla u)$ is the p-Laplacian operator, and $1 < p < N$, $a \geq 0$, $b \geq 0$.

We denote by $W_r^{1,p}(\mathbb{R}^N)$ the space of radially symmetric functions in

$$W^{1,p}(\mathbb{R}^N) = \left\{ u \in L^p(\mathbb{R}^N) : \nabla u \in L^p(\mathbb{R}^N) \right\}.$$

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$W_{r,a}^{1,p}(\mathbb{R}^N)$ is denoted by the space of radially symmetric functions in

$$W_a^{1,p}(\mathbb{R}^N) = \left\{ u \in W^{1,p}(\mathbb{R}^N) : \int_{\mathbb{R}^N} |x|^a |u|^p < \infty \right\}.$$

We also denote by $D_r^{1,p}(\mathbb{R}^N)$ the space of radially symmetric functions in

$$D^{1,p}(\mathbb{R}^N) = \left\{ u \in L^{\frac{Np}{N-p}}\left(\mathbb{R}^N\right) : \nabla u \in L^p\left(\mathbb{R}^N\right) \right\}.$$

Our main results are:

Theorem 1.1. *If $a \geq 0$, $b \geq 0$, $1 < p < N$ and*

$$p < q < \tilde{q} = \frac{Np}{N-p} + \frac{bp}{N-p},$$

$$pb - a\left(p + \frac{(p-1)(q-p)}{p}\right) < (q-p)(N-1),$$

then the problem (1.2) has a radial solution.

Remark 1. In [8], Sirakov proves that the problem (1.2) with $p = 2$ has a solution for

$$2 < q < q^\# = \frac{2N}{N-2} - \frac{4b}{a(N-2)}.$$

In [6], P. Sintzoff and M. Willem proved the existence of a solution of the problem (1.2) with

$$p = 2, \ q \leq 2^*, \quad 2b - a\left(1 + \frac{q}{2}\right) < (N-1)(q-2).$$

Theorem 1.1 extends the results of [6] to the general equation with a $p-$Laplacian operator.

Radial Solution and
Nonradial Solutions

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Theorem 1.2. *Suppose that $a \geq 0$, $b \geq 0$, $1 < p < N$ and*

$$p < q < \frac{Np}{N-p},$$

$$pb - a\left(p + \frac{(p-1)(q-p)}{p}\right) < (q-p)(N-1), \quad aq < pb,$$

then for every R, problem (1.2) with $\Omega = B(0,R)$, R large enough has a radial and a nonradial solution.

This paper is organized as follows: In Section 2, we study (1.2) in the case of $\Omega = \mathbb{R}^N$. We prove the existence of a radial least energy solution of (1.2) when

$$1 < p < N, \; p < q < \tilde{q}, \quad pb - a\left(p + \frac{(p-1)(q-p)}{p}\right) < (q-p)(N-1).$$

In Section 3, we consider the existence of nonradial solutions of (1.2) with $\Omega = B(0,R)$, R large enough. Finally, in Section 4, we consider necessary conditions for the existence of solutions of (1.2).

2. Radial Solution

In this paper, unless stated otherwise, all integrals are understood to be taken over all of $\mathbb{R}^N$. Also, throughout the paper, we will often denote various constants by the same letter.

Lemma 2.1. *Suppose that* $1 < p < N$. *There exist* $A_N > 0$, *such that, for every* $u \in W_{r,a}^{1,p}(\mathbb{R}^N)$, $u \in C(\mathbb{R}^N \backslash \{0\})$, *for* $a \geq \frac{p}{p-1}(1-N)$, *we have that*

$$|x|^{\frac{N-1}{p}+\frac{a(p-1)}{p^2}} |u(x)| \leq A_N \left(\int |x|^a |u|^p \right)^{\frac{p-1}{p^2}} \left(\int |\nabla u|^p \right)^{\frac{1}{p^2}}.$$

Proof. Since

$$\frac{d}{dr}\left(|u|^p r^{a \cdot \frac{p-1}{p}} r^{N-1} \right) = \frac{p}{2} \left(|u|^2 \right)^{\frac{p}{2}-1} \cdot 2u \cdot \frac{du}{dr} r^{a \cdot \frac{p-1}{p}} r^{N-1}$$
$$+ |u|^p \left(a \cdot \frac{p-1}{p} + N - 1 \right) r^{a \cdot \frac{p-1}{p}-1} r^{N-1},$$

and

$$a \geq \frac{p}{p-1}(1-N),$$

we get that

$$\frac{d}{dr}\left(|u|^p r^{a \cdot \frac{p-1}{p}} r^{N-1} \right) \geq pu|u|^{p-2}\frac{du}{dr} r^{a \cdot \frac{p-1}{p}} r^{N-1}$$

and obtain

$$r^{a \cdot \frac{p-1}{p}} r^{N-1} |u(r)|^p \leq A_N \int_r^{+\infty} |u|^{p-1} \left| \frac{du}{dr} \right| S^{N-1} S^{a \cdot \frac{p-1}{p}} dS$$
$$\leq A_N \int |u|^{p-1} \left| \frac{du}{dr} \right| |x|^{a \cdot \frac{p-1}{p}} dx$$

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$$\leq A_N \left(\int |x|^a |u|^p \right)^{\frac{p-1}{p}} \left(\int |\nabla u|^p \right)^{\frac{1}{p}}.$$

It follows that

$$|x|^{N-1+a \cdot \frac{p}{p-1}} |u(x)|^p \leq A_N \left(\int |x|^a |u|^p \right)^{\frac{p-1}{p}} \left(\int |\nabla u|^p \right)^{\frac{1}{p}},$$

and we have

$$|x|^{\frac{N-1}{p} + \frac{a(p-1)}{p^2}} |u(x)| \leq A_N \left(\int |x|^a |u|^p \right)^{\frac{p-1}{p^2}} \left(\int |\nabla u|^p \right)^{\frac{1}{p^2}}.$$

$\square$

Lemma 2.2. *If* $1 < p < N$, $p \leq r < \frac{pN}{N-p}$, *then for any* $u \in W^{1,p}(\mathbb{R}^N)$, *we have that*

$$\int |u|^r dx \leq C \left(\int |\nabla u|^p \right)^{\frac{N(r-p)}{p^2}} \left(\int |u|^p \right)^{\frac{Np+r(p-N)}{p^2}}.$$

Proof. The proof can be adapted directly from the Gagliardo-Nirenberg inequality.

$\square$

The following inequality extends the results of [5] to the general equation with the $p-$Laplacian operator.

Lemma 2.3. *For*

$$1 < p < N, \quad p < q < \frac{pN}{N-p} + \frac{c}{\frac{N-1}{p} + \frac{a(p-1)}{p^2}}, \quad a \geq \frac{p}{p-1}(1-N),$$

there exist $B_{N,p,c}$ such that for every $u \in D_r^{1,p}(\mathbb{R}^N)$, we have

$$\int |x|^c |u|^q dx \le B_{N,p,c} \left(\int |\nabla u|^p \right)^{\frac{c}{p(N-1)+a(p-1)} + \frac{N}{p^2}\left(q - p - \frac{cp^2}{p(N-1)+a(p-1)} \right)}.$$

Proof. Using Lemma 2.1 and Lemma 2.2, we have

$$\int |x|^c |u|^q dx$$

$$= \int \left(|x|^{\frac{N-1}{p} + \frac{a(p-1)}{p^2}} \right)^{\frac{c}{(N-1)/p + a(p-1)p^{-2}}} \left(|u| \right)^{\frac{c}{(N-1)/p + a(p-1)p^{-2}}} \left(|u| \right)^{q - \frac{c}{(N-1)/p + a(p-1)p^{-2}}} dx$$

$$\le \left(\int |x|^a |u|^p \right)^{\frac{p-1}{p^2} \cdot \frac{c}{(N-1)/p + a(p-1)p^{-2}}} \left(\int |\nabla u|^p \right)^{\frac{1}{p^2} \cdot \frac{c}{(N-1)/p + a(p-1)p^{-2}}}$$

$$\cdot \left(\int |\nabla u|^p \right)^{\frac{N}{p^2}\left(q - p - \frac{c}{(N-1)/p + a(p-1)p^{-2}} \right)} \left(\int |u|^p \right)^{\frac{Np}{p^2} + \frac{p-N}{p^2}\left(q - \frac{c}{(N-1)/p + a(p-1)p^{-2}} \right)}$$

$$= \left(\int |x|^a |u|^p dx \right)^{\frac{c(p-1)}{p(N-1)+a(p-1)}} \left(\int |u|^p \right)^{\frac{Np}{p^2} + \frac{p-N}{p^2}\left(q - \frac{cp^2}{p(N-1)+a(p-1)} \right)}$$

$$\cdot \left(\int |\nabla u|^p \right)^{\frac{c}{p(N-1)+a(p-1)} + \frac{N}{p^2}\left(q - p - \frac{cp^2}{p(N-1)+a(p-1)} \right)}$$

$$\le B_{N,p,c} \left(\int |\nabla u|^p \right)^{\frac{c}{p(N-1)+a(p-1)} + \frac{N}{p^2}\left(q - p - \frac{cp^2}{p(N-1)+a(p-1)} \right)}.$$

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Next, to prove Theorem 1.1, we consider the following minimization problem

$$m = m(a,b,p,q) = \inf_{\substack{u \in W_{r,a}^{1,p}(\mathbb{R}^N) \\ \int |x|^b |u|^q dx = 1}} \int \left(|\nabla u|^p + |x|^a |u|^p \right) dx.$$

Theorem 2.4. *If $a \geq 0,\ b \geq 0,\ 1 < p < N$ and*

$$p < q < \tilde{q} = \frac{Np}{N-p} + \frac{bp}{N-p},$$

$$pb - a\left(p + \frac{(p-1)(q-p)}{p} \right) < (q-p)(N-1),$$

then $m(a,b,p,q)$ is achieved.

Proof. Let $(u_n) \subset W_{r,a}^{1,p}(\mathbb{R}^N)$ be a minimizing sequence for $m = m(a,b,p,q)$:

$$\int |x|^b |u_n|^q dx = 1,$$

$$\int \left(|\nabla u_n|^p + |x|^a |u_n|^p \right) dx \to m.$$

By going (if necessary) to a subsequence, we can assume that $u_n \rightharpoonup u$ in $W_{r,a}^{1,p}(\mathbb{R}^N)$. Hence, by weak lower semicontinuity, we have

$$\int \left(|\nabla u|^p + |x|^a |u|^p \right) dx \leq m,$$

$$\int |x|^b |u|^q dx \leq 1.$$

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If c is defined by $q = \frac{pN}{N-p} + \frac{pc}{N-p}$, then $c < b$ and it follows from Lemma 2.3 that

$$\int_{|x|\leq\varepsilon} |x|^b |u_n|^q dx \leq \varepsilon^{b-c} \int |x|^c |u_n|^q dx \leq C\varepsilon^{b-c}.$$

Since (u_n) is bounded in $W_{r,a}^{1,p}(\mathbb{R}^N)$. We deduce from Lemma 2.1 that

$$\int_{|x|\geq\frac{1}{\varepsilon}} |x|^b |u_n|^q dx = \int_{|x|\geq\frac{1}{\varepsilon}} |x|^{b-a} |u_n|^{q-p} |x|^a |u_n|^p dx$$

$$\leq \left(\frac{1}{\varepsilon}\right)^{b-a-(q-p)(\frac{N-1}{p}+\frac{a(p-1)}{p^2})} C \int |x|^a |u|^p dx$$

$$\leq C\varepsilon^{a(\frac{q+1}{p}-\frac{q}{p^2})-b+\frac{(q-p)(N-1)}{p}}.$$

So we get that, for every $t < 1$, there exists $\varepsilon > 0$, such that for every n,

$$\int_{\varepsilon\leq|x|\leq\frac{1}{\varepsilon}} |x|^b |u_n|^q dx \geq t.$$

By the Rellich theorem and Lemma 2.1,

$$1 \geq \int |x|^b |u_n|^q dx \geq \int_{\varepsilon\leq|x|\leq\frac{1}{\varepsilon}} |x|^b |u_n|^q dx \geq t.$$

Finally $\int |x|^b |u|^q dx = 1$ and $m = m(a,b,p,q)$ is achieved at u. $\qquad\square$

Now we will prove Theorem 1.1.

Proof. By Theorem 2.4, m is achieved. Then by the Lagrange multiplier rule, the symmetric criticality principle (see e.g. [13]) and the maximum principle, we obtain

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a solution of

$$
\begin{cases}
-\Delta_p v + |x|^a |v|^{p-2} v = \lambda |x|^b |v|^{q-2} v, \\[2mm]
v > 0, \quad v \in W^{1,p}(\mathbb{R}^N).
\end{cases}
$$

Hence $u = \lambda^{\frac{1}{q-p}} v$ is a radial solution of (1.2), with $\lambda = \frac{p}{q} m$. $\qquad\square$

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3. Nonradial Solutions

In this section, we will prove Theorem 1.2. We use the preceding results to construct nonradial solutions of problem (1.2) in the case $\Omega = B(0, R)$.

Consider

$$M = M(a, b, p, q) = \inf_{\substack{u \in W_a^{1,p}(\mathbb{R}^N) \\ \int |x|^b |u|^q dx = 1}} \int \left(|\nabla u|^p + |x|^a |u|^p \right) dx.$$

It is clear that $M \leq m$, and using our previous results, we prove that M is achieved under some conditions.

Theorem 3.1. *If $a \geq 0$, $b \geq 0$, $1 < p < N$ and*

$$p < q < q^\# = \frac{pN}{N-p} - \frac{p^2 b}{a(N-p)},$$

then $M(a, b, p, q)$ is achieved.

Proof. Let $(u_n) \subset W_a^{1,p}(\mathbb{R}^N)$ be a minimizing sequence for $M = M(a, b, p, q)$:

$$\int |x|^b |u_n|^q dx = 1,$$

$$\int \left(|\nabla u_n|^p + |x|^a |u_n|^p \right) dx \to M.$$

By going (if necessary) to a subsequence, we can assume that $u_n \rightharpoonup u$ in $W_a^{1,p}(\mathbb{R}^N)$.

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Hence, by weak lower semicontinuity, we have

$$\int \left(|\nabla u|^p + |x|^a |u|^p \right) dx \le M,$$

$$\int |x|^b |u|^q dx \le 1.$$

If c is defined by $q = \frac{pN}{N-p} - \frac{p^2 c}{a(N-p)}$, then $c > b$ and

$$r = \frac{a}{c}, \qquad s = \frac{\frac{aNp}{N-p}}{aq - pc}$$

are conjugate. It follows from the Hölder and Sobolev inequalities that

$$\int_{|x| \ge \frac{1}{\varepsilon}} |x|^b |u_n|^q dx \le \left(\frac{1}{\varepsilon} \right)^{b-c} \int |x|^c |u_n|^q dx$$

$$= \left(\frac{1}{\varepsilon} \right)^{b-c} \int |x|^c |u_n|^{\frac{pc}{a}} |u_n|^{q - \frac{pc}{a}} dx$$

$$\le \varepsilon^{c-b} \left(\int |x|^a |u_n|^p dx \right)^{\frac{1}{r}} \left(\int |u_n|^{\frac{Np}{N-p}} dx \right)^{\frac{1}{s}}$$

$$\le C \varepsilon^{c-b}.$$

As in Theorem 2.4, for every $t < 1$, there exists $\varepsilon > 0$ such that, for every n,

$$\int_{|x| \le \frac{1}{\varepsilon}} |x|^b |u_n|^q dx \ge t.$$

By the compactness of the Sobolev theorem in the bounded domain, for $1 < p <$

N, $p < q < \frac{Np}{N-p}$,

$$1 \geq \int |x|^b |u|^q dx \geq \int_{|x| \leq \frac{1}{\varepsilon}} |x|^b |u_n|^q dx \geq t.$$

Hence $\int |x|^b |u|^q dx = 1$ and $M = M(a, b, p, q)$ is achieved at u. $\qquad\square$

Now we will prove Theorem 1.2.

Proof. By Theorem 2.4, $m(a, b, p, q)$ is positive. Since $pb > aq$, it is easy to verify that $M(a, b, p, q) = 0$. Let us define

$$M(a, b, p, q, R) = \inf_{\substack{u \in W_a^{1,p}(B(0,R)) \\ \int_{B(0,R)} |x|^b |u|^q dx = 1}} \int_{B(0,R)} \left(|\nabla u|^p + |x|^a |u|^p \right) dx,$$

$$m(a, b, p, q, R) = \inf_{\substack{u \in W_{r,a}^{1,p}(B(0,R)) \\ \int_{B(0,R)} |x|^b |u|^q dx = 1}} \int_{B(0,R)} \left(|\nabla u|^p + |x|^a |u|^p \right) dx.$$

It is clear that, for every $R > 0$, $M(a, b, p, q, R)$ and $m(a, b, p, q, R)$ are achieved and

$$\lim_{R \to \infty} M(a, b, p, q, R) = M(a, b, p, q) = 0,$$

$$\lim_{R \to \infty} m(a, b, p, q, R) = m(a, b, p, q) > 0.$$

Then from Theorem 1.1, we know that problem (1.2) with $B(0, R)$ has a radial solution.

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On the other hand, by the Lagrange multiplier rule, the symmetric criticality principle (see e.g.[13]) and the maximum principle, we obtain a solution of

$$
\begin{cases}
-\Delta_p v + |x|^a |v|^{p-2} v = \lambda |x|^b |v|^{q-2} v & \text{in } B(0, R) \\
v > 0, \ \ u \in W^{1,p}(B(0, R)), \\
v = 0, & \text{on } \partial B(0, R).
\end{cases}
$$

Hence $u = \lambda^{\frac{1}{q-p}} v$ is a solution of (1.2), with $\lambda = \frac{p}{q} M(a, b, p, q, R)$. Thus, Problem (1.2) has a nonradial solution. □

4. Necessary Conditions

In this section we obtain a nonexistence result for the solution of problem (1.2) using a Pohozaev-type identity. The Pohozaev identity has been derived for very general problems by H. Egnell [3].

Lemma 4.1. *Let $u \in W^{1,p}(\mathbb{R}^N)$ be a solution of (1.2), then u satisfies*

$$\frac{N-p}{p}\int |\nabla u|^p dx + \frac{N+a}{p}\int |x|^a |u|^p dx - \frac{N+b}{q}\int |x|^b |u|^q dx = 0.$$

Theorem 4.2. *Suppose that*

$$\tilde{q} = \frac{Np}{N-p} + \frac{pb}{N-p} \leq q$$

or

$$\frac{N+a}{p} \leq \frac{N+b}{q}.$$

Then there is no solution for problem (1.2).

Proof. Multiplying (1.2) by u and integrating, we see that

$$\int |x|^b |u|^q dx = \int \left(|\nabla u|^p + |x|^a |u|^p \right) dx.$$

On the other hand, using Lemma 4.1, we obtain

$$\left(\frac{N-p}{p} - \frac{N+b}{q} \right)\int |\nabla u|^p dx + \left(\frac{N+a}{p} - \frac{N+b}{q} \right)\int |x|^a |u|^p dx = 0.$$

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So, if u is a solution of problem (1.2), we must have

$$\frac{N-p}{p} < \frac{N+b}{q}, \qquad \frac{N+a}{p} > \frac{N+b}{q}.$$

$\square$

Remark 2. The second assumption of Theorem 2.4,

$$pb - a\left(p + \frac{(q-p)(p-1)}{p}\right) < (q-p)(N-1)$$

implies that

$$\frac{N+b}{q} < \frac{N+a}{p}.$$

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