Lobachevskii Journal of Mathematics
http://ljm.ksu.ru
Vol. 18, 2005, 139–149

Ghulam Mustafa, Nusrat Anjum Noshi and Abdur Rashid

SOME RANDOM COINCIDENCE AND RANDOM FIXED POINT THEOREMS FOR HYBRID CONTRACTIONS

(submitted by D. Kh. Mushtari)

Abstract. Some new random coincidence point and random fixed point theorems for multifunctions in separable complete metrically convex metric spaces are proved. Our results are stochastic generalizations of classical coincidence and fixed point theorems.

1. Introduction

In order to give stochastic generalizations for classical coincidence point theorems and classical fixed point theorems many authors ([1, 3, 4, 5, 6, 8, 9]) introduced more general contractive inequalities. We consider a class of generalized contractions that includes the classes considered in ([1, 3, 4, 5, 6, 8, 9]) and this enables us to prove more general random fixed point and random coincidence point theorems for multifunctions. The results presented in this paper are stochastic versions of corresponding results in [10].

Throughout this paper (X, d) is a separable complete metrically convex metric space, K is a nonempty subset of $X = (X, d)$ and (Ω, σ) is measurable space with a σ-algebra σ of subsets of Ω. Let 2^K be the family of all subsets of K, and $CB(X)$ the family of all nonempty closed

---

2000 Mathematical Subject Classification. 54H25,47H10.
Key words and phrases. multifunction, random fixed point, random coincidence point.

bounded subsets of X. For any nonempty subsets A, B of X, we write

$$d(x, A) = inf\{d(x, a) : a \in A\}(x \in X),$$

$$d(A, B) = inf\{d(a, b) : a \in A, b \in B\},$$

$$H(A, B) = max\left\{\sup_{a \in A} d(a, B), \sup_{b \in B} d(b, A)\right\},$$

and $H(.,.)$ is called the Hausdorff metric on $CB(X)$.

Definition 2.1 A mapping $\mu : \Omega \to 2^K$ is called *measurable* if for any open subset C of K, $\mu^{-1}(C) = \{w \in \Omega : \mu(w) \cap C \neq \emptyset\} \in \sigma$.

Definition 2.2 A mapping $z : \Omega \to X$ is said to be a *measurable selector* of a measurable mapping $\mu : \Omega \to 2^K$ if z is measurable and for any $w \in \Omega, z(w) \in \mu(w)$.

Definition 2.3 A metric space (X, d) is said to be *metrically convex* if for any $x, y \in X$ with $x \neq y$, there exists $z \in X$, $x \neq z \neq y$ such that

$$d(x, z) + d(z, y) = d(x, y)$$

Definition 2.4 A mapping $T : \Omega \times K \to X$ is called a *random operator* if for any $x \in K, T(., x)$ is measurable. A mapping $F : \Omega \times K \to CB(X)$ is called a *multifunction* if for every $x \in K, F(., x)$ is measurable.

Definition 2.5 A measurable mapping $z : \Omega \to X$ is called a *random fixed point* of a multifunction (*random operator*) $F : \Omega \times K \to CB(X)$ $(T : \Omega \times K \to X)$ if for every $w \in \Omega, z(w) \in F(w, z(w))$ $(T(w, z(w)) = z(w))$.

Definition 2.6 A measurable mapping $z : \Omega \to X$ is a *random coincidence point* of $F : \Omega \times K \to CB(X)$ and $T : \Omega \times K \to X$ if for every $w \in \Omega, T(w, z(w)) \in F(w, z(w))$. Let $C(T, F)$ stand for the set of random coincidence points of the maps T and F, that is, $C(T, F)$ $=\{z(w) : T(w, z(w)) \in F(w, z(w))\}$.

Definition 2.7 Let $T : \Omega \times K \to X$ be a random operator and $F : \Omega \times K \to CB(X)$ be a multifunction. Then T and F will be called pointwise $R-$weakly commuting on K if given $x \in K$ and $T(w, x) \in K$, there exists a measurable map $R : \Omega \to (0, \infty)$ such that for each $y \in K \cap F(w, x)$,

$$d(T(w, y), F(w, T(w, x))) \leq R(w)d(F(w, x), T(w, x)). \tag{*}$$

The maps T and F will be called $R-$weakly commuting on K if for each $x \in K$, $T(w, x) \in K$ and (*) holds. If $R(w) = 1$ for each $w \in \Omega$, we get the definition of weak commutativity of F and T on K. T and F are commuting at a point $x \in K$ if $T(w, F(w, x)) \subset F(w, T(w, x))$ whenever

$F(w,x) \subset K$ and $T(w,x) \in K$. T and F are commuting on K if they are commuting at each point $x \in K$.

2. MAIN RESULTS

Let $F, G : \Omega \times K \to CB(X)$ be multifunctions and $S, T : \Omega \times K \to X$ be random operators such that

$$\begin{aligned} H(F(w,x), G(w,y)) \leq \alpha(w) d(T(w,x), S(w,y)) \\ + \beta(w)[d(T(w,x), F(w,x)) + d(S(w,y), G(w,y))] \\ + \gamma(w)[d(T(w,x), G(w,y)) + d(S(w,y), F(w,x))] \quad (2.1) \end{aligned}$$

for each $x, y \in K$ and for each $w \in \Omega$, where $\alpha, \beta, \gamma : \Omega \to (0, \infty)$ are measurable mappings such that

$$(\alpha(w) + \beta(w) + \gamma(w))(1 + \beta(w) + \gamma(w))/(1 - \beta(w) - \gamma(w))^2 < 1. \quad (2.2)$$

Theorem 1. Let (X,d) be a separable complete metrically convex metric space, K a nonempty closed subset of X, and δK the boundary of K. Let $F, G : \Omega \times K \to CB(X)$ be continuous multifunctions and $S, T : \Omega \times K \to X$ be random operators, such that
(i) contractive inequalities (2.1) and (2.2);
(ii) $\delta K \subset S(w,K) \cap T(w,K)$, $F(w,K) \cap K \subset S(w,K)$, $G(w,K) \cap K \subset T(w,K)$; and
(iii) $T(w,x) \in \delta K \Rightarrow F(w,x) \subset K$, $S(w,x) \in \delta K \Rightarrow G(w,x) \subset K$;
are satisfied.
If, either $T(w,K)$ and $S(w,K)$ or $F(w,K)$ and $G(w,K)$ are closed subspaces of X, then
(a) F and T have a random coincidence point;
(b) G and S have a random coincidence point.
Furthermore,
(c) F and T have a common random fixed point $T(w, v(w))$ provided $T(w, T(w, v(w))) = T(w, v(w))$ and T and F are commuting at $v(w) \in C(T,F)$;
(d) G and S have a common random fixed point $S(w, \mu(w))$, provided $S(w, S(w, \mu(w))) = S(w, \mu(w))$ and S and G are commuting at $\mu(w) \in C(S,G)$;
(e) S, T, F and G have a common random fixed point, provided (c) and (d) both are true.

Proof. If the following equality

$$t(w) = (\alpha(w) + \beta(w) + \gamma(w))(1 + \beta(w) + \gamma(w))/(1 - \beta(w) - \gamma(w))^2 = 0$$

holds true, then the theorem holds trivially. Next if $t(w) > 0$, then we proceed to construct the sequences $\{x_n(w)\}$ and $\{y_n(w)\}$, where $x_n, y_n : \Omega \to X$ are measurable mappings.
Let $x_0, x_1 : \Omega \to X$ be a measurable mappings such that $y_1 : \Omega \to X$ defined by $y_1(w) = S(w, x_1(w)) \in F(w, x_0(w))$, for all $w \in \Omega$. Indeed, since F is a continuous random operator, we conclude that, for every $v \in X$, the map $(w, x) \to d(v, F(w, x))$ is a caratheodory function (that is measurable in $w \in \Omega$, continuous in $x \in X$). Thus it is jointly measurable. Hence since $x_0 : \Omega \to X$ is measurable, $w \to d(v, F(w, x_0(w)))$ is measurable. Therefore, $w \to F(w, x_0(w))$ is weakly measurable by Wagner ([4], p 868). By Kuratowski, K ([7], selection theorem 8), there exists a measurable map $x_1 : \Omega \to X$ such that $y_1(w) = S(w, x_1(w)) \in F(w, x_0(w))$ for $x_0(w), x_1(w) \in K$, for all $w \in \Omega$. It follows from (ii) and (iii) that $F(w, x_0(w)) \subset K$. Therefore, $y_1(w) \in K$. It further implies by Itoh ([9], Proposition 4), (ii) and (iii) that there exist measurable mappings $x_2, y_2 : \Omega \to X$ such that, for each $w \in \Omega$, and for $y_2(w) \in K$ (suppose), we have $x_2(w) \in K$ and $y_2(w) = T(w, x_2(w)) \in G(w, x_1(w))$ such that

$$d(y_1(w), y_2(w)) \le H(F(w, x_0(w)), G(w, x_1(w)))+ \\ ((1 - \beta(w) - \gamma(w))/(1 + \beta(w) + \gamma(w)))t(w).$$

If $y_2(w) \notin K$, then there exists a measurable map $p : \Omega \to X$ such that $p(w) \in \delta K$ and

$$d(S(w, x_1(w)), p(w)) + d(p(w), y_2(w)) = d(S(w, x_1(w)), y_2(w)).$$

Since $p(w) \in \delta K \subset T(w, K)$, there exists $x_2(w) \in K$ such that $p(w) = T(w, x_2(w))$ and so

$$d(S(w, x_1(w)), T(w, x_2(w)))+d(T(w, x_2(w)), y_2(w)) = d(S(w, x_1(w)), y_2(w)).$$

Thus repeating the above arguments, we obtain two sequences $\{x_n(w)\}$ and $\{y_n(w)\}$, where $x_n, y_n : \Omega \to X$ are measurable mappings, and $x_n(w) \in K$ such that

- $y_{2n}(w) \in G(w, x_{2n-1}(w))$, $y_{2n+1}(w) \in F(w, x_{2n}(w))$,
- $y_{2n}(w) \in K \Rightarrow y_{2n}(w) = T(w, x_{2n}(w))$ or $y_{2n}(w) \notin K$ $\Rightarrow T(w, x_{2n}(w)) \in \delta K$ and

$$d(S(w, x_{2n-1}(w)), T(w, x_{2n}(w))) + d(T(w, x_{2n}(w)), y_{2n}(w)) \\ = d(S(w, x_{2n-1}(w)), y_{2n}(w)),$$

- $y_{2n+1}(w) \in K$, $y_{2n+1}(w) = S(w, x_{2n+1}(w))$, or $y_{2n+1}(w) \notin K$, $S(w, x_{2n+1}(w)) \in \delta K$, and

$$d(T(w, x_{2n}(w)), S(w, x_{2n+1}(w))) + d(S(w, x_{2n+1}(w)), y_{2n+1}(w)) \\ = d(T(w, x_{2n}(w)), y_{2n+1}(w)).$$

$$d(y_{2n-1}(w), y_{2n}(w)) \leq H(G(w, x_{2n-1}(w)), F(w, x_{2n-2}(w))) \\ + ((1 - \beta(w) - \gamma(w))/(1 + \beta(w) + \gamma(w)))t^{2n-1}(w),$$

$$d(y_{2n}(w), y_{2n+1}(w)) \leq H(F(w, x_{2n}(w)), G(w, x_{2n-1}(w))) \\ + ((1 - \beta(w) - \gamma(w))/(1 + \beta(w) + \gamma(w)))t^{2n}(w).$$

Put

$$P_0 = \{T(w, x_{2i}(w)) \in \{T(w, x_{2n}(w))\} : T(w, x_{2i}(w)) = y_{2i}(w)\},$$

$$P_1 = \{T(w, x_{2i}(w)) \in \{T(w, x_{2n}(w))\} : T(w, x_{2i}(w)) \neq y_{2i}(w)\},$$

$$Q_0 = \{S(w, x_{2i+1}(w)) \in \{S(w, x_{2n+1}(w))\} : S(w, x_{2i+1}(w)) = y_{2i+1}(w)\},$$

$$Q_1 = \{S(w, x_{2i+1}(w)) \in \{S(w, x_{2n+1}(w))\} : S(w, x_{2i+1}(w)) \neq y_{2i+1}(w)\}.$$

Further, as shown in [2], for measurable maps $z_n : \Omega \to X$, $\{z_n(w)\}$ is a Cauchy sequence, where

$$z_{2n}(w) = T(w, x_{2n}(w)), \quad z_{2n+1}(w) = S(w, x_{2n+1}(w)),$$

and there exists at least one subsequence

$$\{T(w, x_{2n_k}(w))\}, \text{ or } \{S(w, x_{2n_k+1}(w))\},$$

which is contained in P_0, or Q_0, respectively. First we suppose that there exists a subsequence $\{T(w, x_{2n_k}(w))\}$ which is contained in P_0, and $T(w, K)$, $S(w, K)$ are closed subspaces of X. Since $\{T(w, x_{2n_k}(w))\}$ is a Cauchy sequence in $T(w, K)$. Then there exists a measurable map $u : \Omega \to X$ such that $\{T(w, x_{2n_k}(w))\} \to u(w) \in T(w, K)$. Let $v(w) \in K$ for a measurable map $v : \Omega \to X$ and $(w, v(w)) \in T^{-1}(u(w))$. Then $u(w) = T(w, v(w))$. Since $\{S(w, x_{2n_k+1}(w))\}$ is a subsequence of the Cauchy sequence $\{z_n(w)\}$, $\{S(w, x_{2n_k+1}(w))\}$ converges to $u(w)$ as well.

By (2.1), we have

$$\begin{aligned}
d(F(w,v(w)),T(w,x_{2n_k}(w))) &\le H(F(w,v(w)),G(w,x_{2n_k-1}(w))) \\
&\le \alpha(w)d(T(w,v(w)),S(w,x_{2n_k-1}(w))) \\
&+\beta(w)[d(T(w,v(w)),F(w,v(w)))+d(S(w,x_{2n_k-1}(w)),G(w,x_{2n_k-1}(w)))] \\
&+\gamma(w)[d(T(w,v(w)),G(w,x_{2n_k-1}(w)))+d(S(w,x_{2n_k-1}(w)),F(w,v(w)))] \\
&\le \alpha(w)d(u(w),S(w,x_{2n_k-1}(w))) \\
&+\beta(w)[d(u(w),F(w,v(w)))+d(S(w,x_{2n_k-1}(w)),T(w,x_{2n_k}(w)))] \\
&+\gamma(w)[d(u(w),T(w,x_{2n_k}(w)))+d(S(w,x_{2n_k-1}(w)),F(w,v(w)))].
\end{aligned}$$

Letting $n \to \infty$, we obtain

$$d(F(w,v(w)),u(w)) \le (\beta(w)+\gamma(w))d(u(w),F(w,v(w))),$$

proving $u(w) \in F(w,v(w))$, since $F(w,v(w))$ is closed. This proves (a). Since the Cauchy sequence $\{z_n(w)\}$ converges to $u(w) \in K$ and $u(w) \in F(w,v(w))$, $u(w) \in F(w,K) \cap K \subset S(w,K)$, there exists $\mu(w) \in K$ such that $S(w,\mu(w)) = u(w)$, where $\mu : \Omega \to X$ is a measurable map. By (2.1) again, we have

$$\begin{aligned}
d(S(w,\mu(w)),G(w,\mu(w))) &= d(T(w,v(w)),G(w,\mu(w))) \\
&\le H(F(w,v(w)),G(w,\mu(w))) \le \alpha(w)d(T(w,v(w)),S(w,\mu(w))) \\
&+\beta(w)[d(T(w,v(w)),F(w,v(w)))+d(S(w,\mu(w)),G(w,\mu(w)))] \\
&+\gamma(w)[d(T(w,v(w)),G(w,\mu(w)))+d(S(w,\mu(w)),F(w,v(w)))] \\
&= (\beta(w)+\gamma(w))d(S(w,\mu(w)),G(w,\mu(w))),
\end{aligned}$$

this proves (b).

If $F(w,K)$ and $G(w,K)$ are closed subspaces, then

$$u(w) \in F(w,K) \cap K \subset S(w,K) \text{ or } u(w) \in G(w,K) \cap K \subset T(w,K),$$

and the above argument establishes (a) and (b). If we suppose that there exists a subsequence $\{S(w,x_{2n_k+1}(w))\}$ contained in Q_0, and $T(w,K)$, $S(w,K)$ are closed subspaces of X, then, noting that $\{S(w,x_{2n_k+1}(w))\}$ is a Cauchy sequence in $S(w,K)$, an analogous argument establishes (a) and (b).

To prove (c), note that $v(w) \in C(T,F)$ and $u(w) = T(w,v(w))$. From this $T(w,u(w)) = T(w,T(w,v(w))) = T(w,v(w))$, hence $T(w,u(w)) = u(w)$, and from the commutativity of T and F, we derive

$$\begin{aligned}
u(w) = T(w,u(w)) = T(w,T(w,v(w))) &\in T(w,F(w,v(w))) \\
&\subset F(w,T(w,v(w))) = F(w,u(w)).
\end{aligned}$$

Thus $u(w)$ is a common random fixed point of T and F. Similar argument yields $u(w) = S(w, u(w)) \in G(w, u(w))$, proving (d). Now e) is immediate.

Corollary 1. Let (X, d) be a separable complete metrically convex metric space, K a nonempty closed subset of X, and δK the boundary of K. Let $F, G : \Omega \times K \to CB(X)$ be continuous multifunctions and $T : \Omega \times K \to X$ be a random operator, such that

(i) contractive inequality (2.1) with $S = T$, and inequality (2.2);

(ii) $\delta K \subset T(w, K)$, $F(w, K) \cup G(w, K) \cap K \subset T(w, K)$;

(iii) $T(w, x) \in \delta K \Rightarrow F(w, x) \cup G(w, x) \subset K$;

(iv) either $T(w, K)$ or $F(w, K)$ and $G(w, K)$ are closed subspaces of X.

Then, F, G, and T have a common random coincidence point $v(w)$. Furthermore, F, G, and T have a common random fixed point provided $T(w, v(w))$ is a random fixed point of T and T commutes with each of F and G at $v(w)$.

Theorem 2. Let (X, d) be a separable complete metrically convex metric space, K a nonempty closed subset of X, and δK the boundary of K. Let $F, G : \Omega \times K \to CB(X)$ be continuous multifunctions and $S, T : \Omega \times K \to X$ be continuous random operators, such that

(i) contractive inequalities (2.1) and (2.2);

(ii) $\delta K \subset S(w, K) \cap T(w, K)$, $F(w, K) \cap K \subset S(w, K)$, $G(w, K) \cap K \subset T(w, K)$; and

(iii) $T(w, x) \in \delta K \Rightarrow F(w, x) \subset K$, $S(w, x) \in \delta K \Rightarrow G(w, x) \subset K$;

are satisfied.

Suppose that (T, F) and (S, G) are pointwise R-weakly commuting pairs, then

(a) There exists a point $z(w) \in K$ such that $S(w, z(w)) \in G(w, z(w))$ and $T(w, z(w)) \in F(w, z(w))$.

Furthermore,

(b) T and F have a common random fixed point provided

$$T(w, T(w, z(w))) = T(w, z(w));$$

(c) S and G have a common random fixed point provided

$$S(w, S(w, z(w))) = S(w, z(w));$$

(d) S, T, F and G have a common random fixed point provided (b) and (c) both are true.

Proof. Proceeding as in the proof of Theorem 1, we suppose that there exists a subsequence $\{T(w, x_{2n_k}(w))\}$ which is contained in P_0. Further, subsequences $\{T(w, x_{2n_k}(w))\}$ and $\{S(w, x_{2n_k+1}(w))\}$ both converge to a $z(w) \in K$, since K is closed in complete X, where $z : \Omega \to X$ is a measurable map. Since $T(w, x_{2n_k}(w)) \in G(w, x_{2n_k-1}(w)) \cap K$ and $S(w, x_{2n_k-1}(w)) \in K$, the pointwise $R-$weak commutativity of G and S implies

$$d(S(w, T(w, x_{2n_k}(w))), G(w, S(w, x_{2n_k-1}(w)))) \\ \leq R_1(w) d(T(w, x_{2n_k}(w)), S(w, x_{2n_k-1}(w))) \quad (2.3)$$

for some measurable map $R_1 : \Omega \to (0, \infty)$. Also,

$$d(S(w, T(w, x_{2n_k}(w))), G(w, z(w))) \\ \leq d(S(w, T(w, x_{2n_k}(w))), G(w, S(w, x_{2n_k-1}(w)))) \\ + H(G(w, S(w, x_{2n_k-1}(w))), G(w, z(w))) \quad (2.4)$$

Letting $k \to \infty$ in (2.3) and (2.4) and using the continuity of S and T, we obtain

$$d(S(w, z(w)), G(w, z(w))) \leq 0,$$

yielding $S(w, z(w)) \in G(w, z(w))$. Since $y_{2n_k+1}(w) \in F(w, x_{2n_k}(w)) \cap K$ and $T(w, x_{2n_k}(w)) \in K$, the pointwise $R-$weak commutativity of F and T implies

$$d(T(w, y_{2n_k+1}(w)), F(w, T(w, x_{2n_k}(w)))) \\ \leq R_2(w) d(y_{2n_k+1}(w), T(w, x_{2n_k}(w)))$$

for some measurable map $R_2 : \Omega \to (0, \infty)$. Therefore, as previously, the continuity of T and F implies

$$d(T(w, z(w)), F(w, z(w))) \leq 0,$$

proving $T(w, z(w)) \in F(w, z(w))$. This proves (a).
If we suppose that there exists a subsequence $\{S(w, x_{2n_k+1}(w))\}$ contained in Q_0, then analogous argument establishes (a).

If $T(w, T(w, z(w))) = T(w, z(w))$ then $T(w, z(w)) \in K$. Thus $z(w) \in K$ and $T(w, z(w)) \in K \cap F(w, z(w))$. Now using the pointwise $R-$weak commutativity of T and F at $z(w)$, we get

$$d(T(w, T(w, z(w))), F(w, T(w, z(w)))) \leq R_3(w) d(F(w, z(w)), T(w, z(w))),$$

for some measurable map $R_3 : \Omega \to (0, \infty)$, where $T(w, T(w, z(w))) \in F(w, T(w, z(w)))$. This proves (b). A similar argument proves (c). Now (d) is immediate.

Corollary 2. Let (X, d) be a separable complete metrically convex metric space, K a nonempty closed subset of X, and δK the boundary of K. Let $F, G : \Omega \times K \to CB(X)$ be continuous multifunctions and $T : \Omega \times K \to X$ be a continuous random operator, such that

(i) contractive inequality (2.1) with $S = T$ and inequality (2.2);

(ii) $\delta K \subset T(w, K)$, $F(w, K) \cup G(w, K) \cap K \subset T(w, K)$;

(iii) $T(w, x) \in \delta K \Rightarrow F(w, x) \cup G(w, x) \subset K$.

Suppose that T is pointwise $R-$weakly commuting with each of F and G.

Then, F, G, and T have common random coincidence point $z(w)$. Furthermore, F, G, and T have a common random fixed point, provided $T(w, T(w, z(w))) = T(w, z(w))$.

Consider $F, G : \Omega \times K \to CB(X)$ and $T : \Omega \times K \to X$ satisfying

$$H(F(w, x), G(w, y)) < M(x, y) \tag{2.5}$$

when $M(x, y) > 0$, $x, y \in K$, where

$$\begin{aligned} M(x, y) = \alpha(w)d(T(w, x), T(w, y)) \\ + \beta(w)[d(T(w, x), F(w, x)) + d(T(w, y), G(w, y))] \\ + \gamma(w)[d(T(w, x), G(w, y)) + d(T(w, y), F(w, x))] \end{aligned}$$

and $\alpha, \beta, \gamma : \Omega \to (0, \infty)$ are measurable mappings such that

$$0 < (\alpha(w)+\beta(w)+\gamma(w))(1+\beta(w)+\gamma(w))/(1-\beta(w)-\gamma(w))^2 \leq 1. \tag{2.6}$$

Theorem 3. Let (X, d) be a separable complete metrically convex metric space, K a nonempty compact subset of X, and δK the boundary of K. Let $F, G : \Omega \times K \to CB(X)$ be continuous multifunctions and $T : \Omega \times K \to X$ be a continuous random operator, such that

(i) contractive inequalities (2.5) and (2.6);

(ii) $\delta K \subset T(w, K)$, $F(w, K) \cup G(w, K) \cap K \subset T(w, K)$;

(iii) $T(w, x) \in \delta K \Rightarrow F(w, x) \cup G(w, x) \subset K$.

Suppose that T is pointwise R-weakly commuting with each of F and G;

Then, F, G, and T have common random coincidence point $z(w)$.
Furthermore, F, G, and T have a common random fixed point provided $T(w, z(w))$ is a random fixed point of T.

Proof. In view of the last part of Corollary 2, it is enough to show that F, G and T have a common random coincidence point. We claim that $M(x(w), y(w)) = 0$, for some $x(w), y(w) \in K$, where $x, y : \Omega \to X$ are measurable mappings. Otherwise the function

$$q(x(w), y(w)) = H(F(w, x(w)), G(w, y(w)))/M(x(w), y(w))$$

is continuous and satisfies $q(x(w), y(w)) < 1$ for $(x(w), y(w)) \in K \times K$. Since $K \times K$ is compact, there exists $u(w), v(w) \in K$ such that

$$q(x(w), y(w)) \leq q(u(w), v(w)) = \nu(w) < 1$$

for $x(w), y(w) \in K$ and for some measurable map $\nu : \Omega \to (0, 1)$. Consequently,

$$H(F(w, x(w)), G(w, y(w))) \leq \nu(w)M(x(w), y(w)).$$

Further, in view of (2.6), it is a straightforward verification that

$$(\nu(w)\alpha(w) + \nu(w)\beta(w) + \nu(w)\gamma(w))(1 + \nu(w)\beta(w) + \nu(w)\gamma(w))/ \\ (1 - \nu(w)\beta(w) - \nu(w)\gamma(w))^2 < 1.$$

So, by Corollary 2, $T(w, z(w)) \in F(w, z(w)) \cap G(w, z(w))$ for some $z(w) \in K$, and we have $M(z(w), z(w)) = 0$. This contradicts $M(z(w), z(w)) > 0$. Therefore $M(x(w), y(w)) = 0$ for some $x(w), y(w) \in K$, and this implies $T(w, x(w)) \in F(w, x(w))$ and $T(w, x(w)) = T(w, y(w)) \in G(w, y(w))$. If $M(x(w), y(w)) = 0$, then $T(w, x(w)) \in G(w, x(w))$, and if $M(x(w), x(w)) \neq 0$, then (2.5) implies

$$(1 - \beta(w) - \gamma(w))d(T(w, x(w)), G(w, x(w))) \leq 0,$$

yielding $T(w, x(w)) \in G(w, x(w))$. Similarly, in either of the two cases $M(y(w), y(w)) = 0$ and $M(y(w), y(w)) > 0$, $T(w, y(w)) \in F(w, y(w))$. This proves that F, G and T have a common random coincidence point.

References

[1] Adrian Constantin. *A random fixed point theorem for multifunctions.* Stoch. Anal. & Appl. 12(1) (1994), 65-73

[2] A. Ahmad and M. Imdad. *Some common fixed point theorems for mappings and multi-valued mappings.* J. Math. Anal. Appl. 218(1998), 546-560

[3] C. J. Himmelberg. *Measurable relations.* Fund. Math. 87(1975), 53-72

[4] D.H. Wagner. *Survey of measurable selection theorems.* SIAM. J. Control Optim. 15(1977), 859-903

[5] G. Mustafa. *Random fixed point theorems for contractive type multifunctions.* J. Aust. Math. Soc. 78(2005), 211-220.

[6] I. Beg and N. Shahzad. *Random fixed points of random multivalued operators on polish spaces.* Nonlinear Anal. Theor. Meth. Appl. 20(7) (1993), 835-847

[7] K. Kuratowski and C. Ryll Nardzewski. *A general theorem on selectors.* Bull. Acad. Polon. Sci, Ser. Sci. Math. Astronon. Phys. 13(1965), 397-403

[8] N. S. Papageorgiou. *Random fixed point theorems for multifunctions.* Math. Japonica. 29(1984), 93-106

[9] S. Itoh. *A random fixed point theorem for a multivalued contraction mapping.* Pacific. J. Math. 68(1977), 85-90

[10] S. L. Singh and S. N. Mishra. *Coincidences and fixed points of nonself hybrid contractions.* J. Math. Anal. Appl. 256(2001), 486-497

Department of Mathematics, Multan Public School, Multan, Islamia University Bahawal pur, Pakistan, Pakistan

E-mail address: mustafa_rakib@yahoo.com

Department of Mathematics, COMSATS Institute of Information Technology Lahore, Pakistan

Received February 15, 2005; revised version August 29, 2005