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EXTENSION OF LINE-SPLITTING OPERATION FROM GRAPHS TO BINARY MATROIDS

(submitted by M. M. Arslanov)

Abstract. In this paper, we characterize the n-line splitting operation of graphs in terms of cycles of respective graphs and then extend this operation to binary matroids. In matroids, we call this operation an element-set splitting. The resulting matroid is called the es-splitting matroid. We characterize circuits of an es-splitting matroid. We also characterize the es-splitting matroid in terms of matrices. Also, we show that if M is a connected binary matroid then the es-splitting matroid M_X^e is also connected.

1. Introduction

In [9], Slater specified the n-point splitting and n-line splitting operations in graphs in the following way:

Let G be a graph and U be a vertex of G with deg $U \geq 2n-2$. Let G_1 be the graph obtained from G by replacing U by two adjacent vertices U_1 and U_2, and if vertex X is adjacent to U in G, written X adj U, then make X adj U_1 or X adj U_2 (but not both) such that deg $U_1 \geq n$ and deg $U_2 \geq n$ (see Figure 1.1).

Key words and phrases. Binary matroid, n-line splitting, element-set splitting, connected matroid.

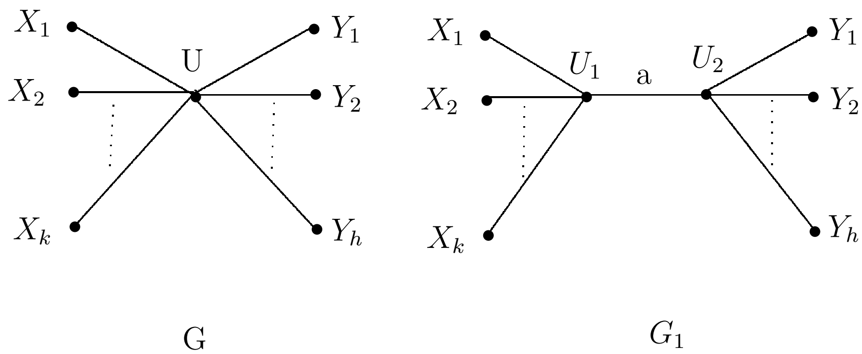

Figure 1.1

We say that G_1 is obtained from G by n-point splitting operation. The operation of n-line splitting is defined as follows.

Let G be a graph and $e = UV$ be an edge of G with deg $U \geq 2n-3$ with U adjacent to $X_1, X_2, \ldots, X_k$, $Y_1, Y_2, \ldots, Y_h$, where h and $k \geq n-2$. Let H be the graph obtained from G by replacing U by two adjacent vertices U_1 and U_2 with V adj U_1, V adj U_2, U_1 adj X_i and U_2 adj Y_j, where $1 \leq i \leq k$ and $1 \leq j \leq h$ and deg $U_1 \geq n$, deg $U_2 \geq n$. Let H be said to arise from G by n-line splitting (see Figure 1.2). We also say that H is a n-line splitting of G.

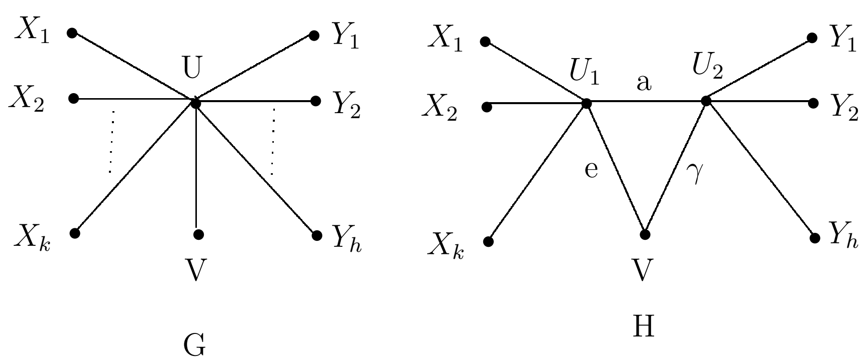

Figure 1.2

The above operations can be related to the earlier splitting operations. Let $X_iU = x_i$ $(1 \leq i \leq k)$ and $UV = e$ and $U_1U_2 = a, U_2V = \gamma$. Moreover, let $X = \{x_1, x_2, \cdots, x_k, e\}$ and $\Delta = \{e, a, \gamma\}$. We denote H by G^e_X. If G_X denote the splitting of G with respect to X (see [6]), then $G^e_X = G_X + a + \gamma$. If G'_X is obtained from G by n-point splitting operation, then $G^e_X = G'_X + \gamma$. In other words, $G^e_X \backslash \{a, \gamma\} = G_X$ and $G^e_X \backslash \{\gamma\} = G'_X$.

In the next proposition, we characterize the cycles of the graph G^e_X in terms of the cycles of G.

In [2] Raghunathan, Shikare and Waphare generalized the splitting operation of graphs to binary matroids. Shikare, Azadi and Waphare [6, 7] extended the notions of n-point splitting from graphs to binary matroids. The authors in [4] determined the bases of splitting matroids and Shikare, Azanchiler and Waphare [8] characterized cocircuits of splitting matroids. We extend the n-line splitting from graphs to binary matroids in next Section. Also we characterize circuits of the result matroid. For the matroid theory we refer the reader to [1, 10].

Proposition 1.1. Let G be a graph and $e = uv$ be an edge of G with deg $u \geq 2n - 3$ and u adjacent to $X_1, X_2, \cdots, X_k, Y_1, Y_2, \cdots, Y_h$, where h and $k \geq n - 2$. Let $X_i u = x_i$ and $Y_j u = y_j$ for $1 \leq i \leq k$; $1 \leq j \leq h$. Let $X = \{e, x_1, x_2, \cdots, x_k\}$ and $Y = \{y_1, y_2, \cdots, y_h\}$. Then C is a cycle in G^e_X if and only if C satisfies one of the following conditions:

(1) C is a cycle in G, containing precisely two elements of X.
(2) C is a cycle in G containing no element of X.
(3) $C = C_1 \cup C_2$, where C_1 and C_2 are edge-disjoint cycles of G, each contains exactly one edge of X and $C_1 \cup C_2$ contains, no cycle of type (1) or (2).
(4) $C = C_1 \cup \{a\}$ when C_1 is a cycle of G, containing precisely one edge of X.
(5) $C = C_1 \cup \{e, \gamma\}$, where C_1 is a cycle of G, containing exactly one element of $X - \{e\}$.
(6) $C = (C_1 \setminus e) \cup \{a, \gamma\}$, where C_1 is a cycle of G, containing precisely one element of $X - e$ and the edge e.
(7) $C = (C_1 \setminus e) \cup \{\gamma\}$, where C_1 is a cycle of G, containing the edge e of X.
(8) $C = \{e, a, \gamma\}$.

2. Splitting of a Binary Matroid with Respect to an Element and a Set

Now, we extend the notion of n-line splitting operation from graphs to binary matroids. In the first step, we consider matrix approach to this operation in binary matroids.

Definition 2.1. Let M be a binary matroid on a set S and X be a subset of $S, e \in X$. Suppose that A is a matrix over $GF(2)$, that represents the matroid M. Let A^e_X be the matrix that is obtained by adjoining an extra row to A with this row being zero everywhere except in the columns corresponding to the elements of X where it takes the value 1, and then adjoining two columns a and γ to the resulting matrix such that the column a is zero everywhere except in the last row (new row) where it takes the value 1, and γ is a sum of two column vectors corresponding to a and e.

Let M_X^e be the vector matroid of the matrix A_X^e. We say that M_X^e has been obtained from M by splitting e and X in M. The transition from M to M_X^e is called splitting of M with respect to e and X. For convenience, we say that M_X^e is a element -set splitting (es-splitting) matroid.

Remark 2.2. Let r and r' be the rank functions of M and M_X^e, respectively. Then $r'(M_X^e) = r(M) + 1$.

In the next proposition we characterize the circuits of the matroid M_X^e.

Theorem 2.3. Let $M = (S, \mathcal{C})$ be a binary matroid, $X \subseteq S, e \in X$ and $a, \gamma \notin S$. Then $M_X^e = (S \cup \{a, \gamma\}, \mathcal{C}_X^e)$, where $\mathcal{C}_X^e = \mathcal{C}_0 \cup \mathcal{C}_1 \cup \mathcal{C}_2 \cup \mathcal{C}_3 \cup \{\Delta\}$ with $\Delta = \{e, a, \gamma\}$ and
$\mathcal{C}_0 = \{C \in \mathcal{C} \mid C$ contains an even number of elements of $X\}$;
$\mathcal{C}_1 =$ The set of minimal members of $\{C_1 \cup C_2 \mid C_1, C_2 \in \mathcal{C}, C_1 \cap C_2 = \phi$ and each C_1 and C_2 contains an odd number of element of X such that $C_1 \cup C_2$ contains no member of $\mathcal{C}_0\}$;
$\mathcal{C}_2 = \{C \cup \{a\} \mid C \in \mathcal{C}$ and C contains an odd number of element of $X\}$.
$\mathcal{C}_3 = \{\ C \cup \{e, \gamma\}, \mid C \in \mathcal{C}, e \notin C$ and C contains odd number of elements of $X\}$.
$\cup \{\ (C \setminus e) \cup \{\gamma\} \mid C \in \mathcal{C}, e \in C$ and C contains odd number of elements of $X\ \}$.
$\cup \{\ (C \setminus e) \cup \{a, \gamma\} \mid C \in \mathcal{C}, e \in C$ and $C \setminus e$ contains odd number of elements of $X\ \}$.

Proof. We prove that $\mathcal{C}_X^e$ satisfies the circuit axioms of a matroid.

(1) Let $A, B \in \mathcal{C}_X^e$, we show that if $A \subseteq B$, then $A = B$. In other words, we prove that $A \not\subset B$ and $B \not\subset A$. The property clearly holds, if both A and B belong to $\mathcal{C}_0, \mathcal{C}_1$ or $\mathcal{C}_2$.

Now, assume that both A and B belong to $\mathcal{C}_3$ and let $A = C_1 \cup \{e, \gamma\}$ and $B = (C_2 \setminus e) \cup \{\gamma\}$, where $C_1, C_2 \in \mathcal{C}, C_1$ and C_2 contain an odd number of element $X, e \notin C_1$ and $e \in \mathcal{C}_2$. Thus $e \in A$ and $e \notin B$, so it follows that $A \not\subset B$. Similarly, $a \notin A$ and $a \in B$, implies that $B \nsubseteq A$.

Next, let $A \in \mathcal{C}_i$ and $B \in \mathcal{C}_j, i \neq j$ and $i, j = 0, 1, 2, 3$.

(i) Let $A \in \mathcal{C}_0$ and $B \in \mathcal{C}_1$. Then $B = C_1 \cup C_2$, where C_1 and C_2 are circuits of M, satisfying the properties stated in the Definition 4.2.1 of $\mathcal{C}_1$.

(ii) Let $A \in \mathcal{C}_0$ and $B \in \mathcal{C}_2$. Then $B = B_1 \cup \{a\}$, where B is a circuit of M, containing an odd number of element of X. Since $A \not\subset B_1$ and $B_1 \not\subset A$, it follows that $A \not\subset B$ and $B \not\subset A$, as desired.

(iii) Suppose $A \in \mathcal{C}_0$ or $\mathcal{C}_1$ and $B \in \mathcal{C}_3$. Since $\gamma \in B$ and $\gamma \notin A$, then $A \not\subset B$ and $B \not\subset A$.

(iv) Let $A \in \mathcal{C}_1$ and $B \in \mathcal{C}_2$. Suppose $A = C_1 \cup C_2$ and $B = B_1 \cup \{a\}$, where C_1, C_2 and B_1 are circuits of M, containing an odd number of element of X. Since $a \in B$ and $a \notin A, B \not\subset A$. Also $A \not\subset B$, for $A \subseteq B$ implies that $A \subseteq B_1$ and $C_1 \cup C_2 \subset B_1$, leads to a contradiction.

(v) Let $A \in \mathcal{C}_2$ and $B \in \mathcal{C}_3$. Then $A = A_1 \cup \{a\}$ and $B = B_1 \cup \{e, \gamma\}$ or $B = (B_2 \setminus e) \cup \{\gamma\}$, where A_1, B_1 and B_2 are circuits in M each contains an odd number of elements of X and $e \notin B_1, e \in B_2$. Since $\gamma \in B$ but $\gamma \notin A$, $B \not\subset A$. Similarly, $A \not\subset B$.

(2) Let $A, B \in \mathcal{C}_X^e$ and $A \neq B$. We prove that there exists $D \in \mathcal{C}_X^e$ such that $D \subseteq A \Delta B$.

Firstly, let $A, B \notin \mathcal{C}_2 \cup \mathcal{C}_3 \cup \{\Delta\}$. By binarity of M,

$$A \Delta B = C_1' \cup C_2 \cup \cdots \cup C_m', \tag{$*$}$$

where $C_1', C_2', \cdots$ and C_m' are disjoint circuits of M. Since A and B each contains an even number of element of X, $A\Delta B$ contains an even number of element of X. If $A\Delta B$ contains no element of X, then each circuit $C_i', i = 1, 2, \cdots, m$ is a member of $\mathcal{C}_0$ and is contained in $A\Delta B$. If for some $j, 1 \leq j \leq m, C_j'$ contains an even number of element of X, then $D = C_j'$. Otherwise, m must be an even integer and for every $j = 1, 2, \cdots, m, C_j'$ must contain an odd number of element of X. If $C_1' \cup C_2'$ contains a member of $\mathcal{C}_0$, say C, then we take $D = C$. Otherwise; $D = C_1' \cup C_2'$ or a minimal member of $\mathcal{C}_1$ contained in it.

Secondly, suppose $A, B \in \mathcal{C}_2 \cup \mathcal{C}_3 \cup \{\Delta\}$, then we have the following cases:

(I) Let $A, B \in \mathcal{C}_2$ and $A = A_1 \cup \{a\}, B = B_1 \cup \{a\}$. Then $A\Delta B = A_1 \Delta B_1 = C_1' \cup C_2' \cup \cdots \cup C_m'$, where $C_1', C_2', \cdots$ and C_m' are disjoint circuits of M. Since each of A and B contains an odd number of elements of $X, A\Delta B$ contains an even number of elements of X. By similar argument as above, we can find $D \in \mathcal{C}_X^e$ such that $D \subseteq A\Delta B$.

(II) Let A and B belong to $\mathcal{C}_3$, we have the following subcases:

(i) $A = A_1 \cup \{e, \gamma\}$ and $B = B_1 \cup \{e, \gamma\}$. (ii) $A = (A_1 \setminus e) \cup \{\gamma\}$ and $B = (B_1 \setminus e) \cup \{\gamma\}$. (iii) $A = A_1 \cup \{e, \gamma\}$, $B = (B_1 \setminus e) \cup \{\gamma\}$. (iv) $A = A_1 \cup \{e, \gamma\}$, $B = (B_1 \setminus e) \cup \{a, \gamma\}$. (v) $A = (A_1 \setminus e) \cup \{a, \gamma\}$ and $B = (B_1 \setminus e) \cup \{a, \gamma\}$. (vi) $A = (A_1 \setminus e) \cup \{a, \gamma\}$, $B = (B_1 \setminus e) \cup \{a, \gamma\}$.

In cases (i), (ii) and (vi), we have $A\Delta B = A_1\Delta B_1 = C'_1 \cup C'_2 \cup \cdots \cup C'_m$, where $C'_1, C'_2, \cdots, C'_m$ are disjoint circuits of M. By the similar arguments as in (1), we can find $D \in \mathcal{C}^e_X$ such that $D \subseteq A\Delta B$. In case (iii), we have $A\Delta B = (A_1\Delta B_1) \cup \{a\}$.

Let $A_1\Delta B_1 = C'_1 \cup C''_2 \cup \cdots \cup C''_n$, where $C''_1, C''_2, \cdots, C''_n$ are disjoint circuits of M. If C''_1 contains an even number of elements of X, then it will be an element of $\mathcal{C}^e_X$ which is contained in $A_1\Delta B_1$, and hence in $A\Delta B$. If C''_1 contains an odd number of elements of X, then $C''_1 \cup \{a\}$ is an element of $\mathcal{C}_2$, contained in $A\Delta B$. By similar argument, the cases (iv) and (v) follow.

(III) Let $A \in \mathcal{C}_2$ and $B \in \mathcal{C}_3$. Then $A = A_1 \cup \{a\}$ and for B, we have two subcases: (1) $B = B_1 \cup \{e, \gamma\}$, and (2) $B = (B_1 \setminus e) \cup \{\gamma\}$. Thus, in (1), we have $A\Delta B = (A_1\Delta B_1) \cup \{e, a, \gamma\}$, where A_1 and B_1 are circuits of M, containing an odd number of element of X. Clearly, $A_1\Delta B_1 \subseteq A\Delta B$. Let $A_1\Delta B_1 = C'_1 \cup C'_2 \cup C'_m$, where $C'_1, C'_2, \cdots, C'_m$ are disjoint circuits of M. By similar arguments as in (2), we can find $D \in \mathcal{C}^e_X$ such that $D \subseteq A_1\Delta B_1$ and hence $D \subseteq A\Delta B$. In (2), we have $A = A_1 \cup \{a\}$ and $B = (B_1 \setminus e) \cup \{\gamma\}$. So $A\Delta B = [(A_1\Delta B_1) \setminus e] \cup \{a, \gamma\}$, where A_1 and B_1 are circuits in M. By the argument as given above, we can find $D \in \mathcal{C}^e_X$, such that $D \subseteq A\Delta B$.

(IV) Let $A = \Delta$ and $B \in \mathcal{C}_2$. Then $B = B_1 \cup \{a\}$, where B_1 is a circuit in M containing an odd number of element of X. We have two subcases: (i) $e \notin B_1$. Then $A\Delta B$ is a circuit of M, containing an even number of elements of X. Thus $D = A\Delta B$. (ii) $e \in B_1$. Then $A\Delta B$ is also a circuit in M^e_X and so $D = A\Delta B$.

(V) Let $A = \Delta$ and $B \in \mathcal{C}_3$. Then $B = B_1 \cup \{e, \gamma\}$ or $B = (B_1 \setminus e) \cup \{\gamma\}$, where B_1 is a circuit of M, containing an odd number of element of X. Since $A\Delta B_1 \subset A\Delta B$, we can find $D = \Delta$, where $D \in \mathcal{C}^e_X$ such that $D \subseteq A\Delta B_1$, and hence $D \subseteq A\Delta B$.

We conclude that $\mathcal{C}^e_X$ is a collection of circuits of a binary matroid on the set $S \cup \{a, \gamma\}$.

$\square$

Definition 2.4. With notation as above, we call the matroid $(S \cup \{a, \gamma\}, \mathcal{C}^e_X)$ as the es-splitting of $M = (S, \mathcal{C})$ and denote it by M^e_X. Thus $M^e_X = (S', \mathcal{C}^e_X)$, where $S' = S \cup \{a, \gamma\}$.

Example 2.5. Consider the matroid $M = F_7$ with ground set $S = \{1,2,3,4,5,6,7\}$ and the set of circuits

$$\begin{aligned}\mathcal{C} &= \{\{1,2,4\},\{1,3,6\},\{2,6,7\},\{4,5,6\},\{1,5,7\},\{3,4,7\},\\ &\quad \{2,3,5\},\{3,5,6,7\},\{2,4,5,7\},\{1,3,4,5\},\{1,2,3,7\},\\ &\quad \{2,3,4,6\},\{1,2,5,7\},\{1,4,6,7\}\}\end{aligned}$$

Let $X = \{1,2,4\}$ and $e = 1$. Then the circuit set of M_X^e is

$$\begin{aligned}\mathcal{C}_X^e &= \{\{3,5,6,7\},\{2,4,5,7\},\{1,3,4,5\},\{1,2,3,7\},\{2,3,4,6\},\\ &\quad \{1,2,5,6\},\{1,4,6,7\},\{1,3,6,a\},\{1,2,4,a\},\{2,6,7,a\},\\ &\quad \{4,5,6,a\},\{1,5,7,a\},\{3,4,7,a\},\{2,3,5,a\},\{1,2,6,7,\gamma\},\\ &\quad \{1,4,5,6,\gamma\},\{1,3,4,7,\gamma\},\{1,2,3,5,\gamma\},\{2,4,8\},\{3,6,\gamma\},\\ &\quad \{5,7,\gamma\},\{1,a,\gamma\},\{3,4,5,a,\gamma\},\{2,3,7,a,\gamma\},\{2,5,6,a,\gamma\},\\ &\quad \{4,6,7,a,\gamma\}\}.\end{aligned}$$

Theorem 2.6. The matrix A_X^e represents the splitting matroid M_X^e.

3. Connectedness of the Splitting Matroid M_X^e

In [5] Shikare characterized the connectedness in splitting matroidM_X. The next theorem characterize connectedness of M_X^e.

Theorem 3.1. Let $M = (S, \mathcal{C})$ be a binary connected matroid. Then M_X^e is connected.

Proof. Let M be a connected matroid on S. Then for every pair $x, y \in S$ there is a circuit of M containing x and y. We show that for any two elements x and y belonging to $S \cup \{a, \gamma\}$, there is a circuit of M_X^e containing x and y. We consider the following cases:

(1) Let $x, y \in \{a, \gamma\}$. Then $x, y \in \Delta$ and we are through.

(2) Suppose $x, y \notin \{a, \gamma\}$. Then $x, y \in S$ and M has a circuit, say C containing x and y. We have the following two subcases:

(i) C contains an even number of elements of X. Then we are through.

(ii) C contains an odd number of elements of X. Then $C \cup \{a\}$ is a circuit of M_X^e containing x and y.

(3) Let $x = a$ and $y \in S$. Then there is a circuit $C \in \mathcal{C}$ such that $y, e \in C$. We have two subcases:

(i) Suppose C contains an even number of elements of X. Then $C \in \mathcal{C}_X^e, e \in C \cap \Delta, y \in C$ and $a \in \Delta$. By [3], there is a circuit say C' of M_X^e such that $a, y \in C'$.

(ii) Let C contains an odd number of elements of X. Then $C \cup \{a\}$ is a circuit in M_X^e and $a, y \in C \cup \{a\}$.

(4) Let $x = \gamma$ and $y \in S$. Then there is a circuit $C \in \mathcal{C}$ such that $y, e \in C$. We consider the following two subcases:

(i) C contains an even number of element of X. Then $C \in \mathcal{C}_X^e, e \in C \cap \Delta, y \in C$ and $\gamma \in \Delta$, so by [3], there is a circuit of M_X^e containing γ and y.

(ii) C contains an odd number of element of X. Then $(C \setminus e) \cup \{\gamma\}$ is a circuit of M_X^e containing γ and y. $\square$

Remark 3.2. Converse of Theorem 3.1 is not true. For example, let M be a cycle matroid of the graph G (See Figure 3.1). Let $X = \{e, x_1, x_2\}$. Then M_X^e is a cycle matroid of G_X^e.

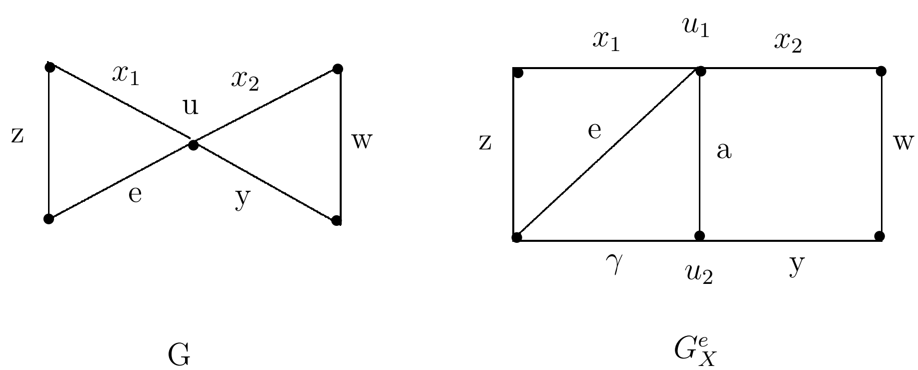

Figure 3.1

We observe that the matroid $M = M(G)$ is disconnected while $M_X^e = M(G_X^e)$ is connected.

The next theorem shows that one can obtain a connected matroid from disconnected matroid with the help of es-splitting operation.

Theorem 3.3. Let M be a bridgeless binary matroid on S with n components $M_1, M_2, \cdots, M_n$. Let x_i be chosen from M_i for $i = 1, 2, \cdots, n$ and $X = \{x_1, x_2, \cdots, x_n\}, e \in X$. Then M_X^e is a connected matroid on $S \cup \{a, \gamma\}$.

Proof. Let $M_1 = (S_1, D_1), M_2 = (S_2, D), \cdots, M_n = (S_n, D_n)$ be the n components of M, where D_i denote the collection of circuits of M_i. Then $S_i \cap S_j = \phi$ for $i \neq j, i, j = 1, 2, \cdots, n$ and $\cup_{i=1}^n S_i = S$. $X = \{x_1, x_2, \cdots, x_n\}$, where $x_i \in S_i$ for $i = 1, 2, \cdots, n$. Since $e \in X$, there is a j such that $x_j = e$, where $1 \leq j \leq n$. Suppose $\mathcal{A}_i = \{C \in D_i \mid x_i \notin C\}$

for $i = 1, 2, \cdots, n$ and $\mathcal{A}_{n+1} = \{C_i \cup C_j \mid C_i \in D_i, C_j \in D_j, x_i \in C_i, x_j \in C_j, i \neq j$ and $i, j = 1, 2, \cdots, n\}$. Let $\mathcal{B}_i = \{C \cup \{a\} \mid C \in D_i, x_i \in C, i = 1, 2, \cdots, n\}$ and

$$\begin{aligned}\mathcal{C}_i &= \{E \cup \{e, \gamma\} \mid E \in D_i, e \notin E, x_i \in E\} \\ &\quad \cup \{(F \setminus e) \cup \{\gamma\} \mid F \in D_i, e \in F, x_i \notin F \setminus \{e\}\} \\ &\quad \cup \{(G \setminus e) \cup \{a, \gamma\} \mid G \in D_i, e \in G, x_i \in G \setminus \{e\}\}.\end{aligned}$$

The matroid M_X^e has the circuit set $\mathcal{C}_X^e$, where $\mathcal{C}_X^e = \mathcal{A}_1 \cup \mathcal{A}_2 \cup \cdots \cup \mathcal{A}_n \cup \mathcal{A}_{n+1} \cup B_1 \cup B_2 \cup \cdots \cup B_n \cup \mathcal{C}_1 \cup \mathcal{C}_2 \cup \cdots \cup \mathcal{C}_n \cup \{\Delta\}$.

Claim. For every pair of elements α, β of $S \cup \{a, \gamma\}$ there is a circuit of M_X^e, containing α and β. We have the following cases:

(I) Let $\alpha, \beta \in S$. Then we consider the following subcases:

(i) Suppose α and β belong to one component, say M_i. Then there is a circuit say C_i of M_i, containing α and β. If x_i does not belong to C_i, then C_i is a circuit of M_X^e, containing α and β. If $x_i \in C_i$, then for any $j \neq i$, C_j is a circuit of M_j containing x_j. Thus $C_i \cup C_j$ is a circuit of M_X^e containing α and β.

(ii) Let α and β belongs to different components of M, say $\alpha \in M_i$ and $\beta \in M_j$. Consider the circuits C_i and C_j, where C_i contains α and x_i and C_j contains β and x_j. Then $C_i \cup C_j$ is a required circuit of M_X^e.

(II) Let $\alpha \in S$ and $\beta \in \{a, \gamma\}$. Then $\alpha \in S_i$ for some i. Consequently, there is a circuit of M_i, say C_i, containing α and x_i. If $\beta = a$ then $C_i \cup \{a\}$ is a circuit of M_X^e containing α and β. If $b = \gamma$, then $(C_i \setminus \{e\}) \cup \{\gamma\}$ is a circuit of M_X^e containing α and β.

(III) If $\alpha = a$ and $\beta = \gamma$, then $\alpha, \beta \in \Delta$, a 3-circuit in M_X^e. □

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