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SELF-SIMILAR SOLUTIONS FOR A NONLINEAR DEGENERATE PARABOLIC EQUATION

(submitted by A. V. Lapin)

Abstract. In this paper, the author investigates the initial boundary value problem for a nonlinear degenerate parabolic equation, which comes from a compressible fluid flowing in a homogeneous isotropic rigid porous medium. We establish the existence of nonnegative self-similar solutions.

1. Introduction

During the past years, many authors have paid much attention to the following equation

$$\frac{\partial u}{\partial t} = \operatorname{div}(|x|^{\alpha}|\nabla(|u|^{m-1}u)|^{N-1}\nabla(|u|^{m-1}u)),$$

in $\mathbb{R}^n$. The equation is a continuous model, which comes from a compressible fluid flowing in a homogeneous isotropic rigid porous medium. The equation is also called non-newtonian polypropic filtration equations. Zhao and Yuan [4], Zhao and Xu [3] consider the Cauchy problem for $\alpha = 0$. They proved the existence of weak solution; see also [6], [7]. After

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introducing the radial variable $r = |x|$, we see that the radial symmetric solution satisfies

$$\frac{\partial u}{\partial t} = \frac{1}{r^{n-1}} \frac{\partial}{\partial r} \left(r^{\alpha+n-1} \left| \frac{\partial}{\partial r}(|u|^{m-1}u) \right|^{N-1} \frac{\partial}{\partial r}(|u|^{m-1}u) \right), \tag{1.1}$$

On the basis of physical consideration, as usual the equation (1.1) is supplemented with the natural boundary value conditions

$$u(0,t) = u_0 t^{\gamma}, \quad u(+\infty, t) = 0, \ t > 0, \tag{1.2}$$

and the initial value condition

$$u(r,0) = 0. \tag{1.3}$$

In this paper, we study the self-similar solution of the problem (1.1)-(1.3). We are seeking solutions of the form

$$u(r,t) = u_0 t^{\gamma} y(x), \quad x = crt^{-\beta}.$$

A direct calculation shows that $y(x)$ should satisfy the following ordinary differential equation

$$\left[x^{\alpha+n-1} \left| (|y|^{m-1}y)' \right|^{N-1} (|y|^{m-1}y)' \right]' = \lambda x^{n-1} y(x) - x^n y'(x), \tag{1.4}$$

where $m > 1, N > 1$, $\beta = \frac{\gamma(mN-1)+1}{N-\alpha+1}$, $C^{N+1} = u_0^{mn-1}\beta, \lambda = \frac{\gamma}{\beta}$, $u_0 > 0$, $\gamma > 0$ with the following initial value conditions

$$y(0) = 1, \quad \lim_{x \to +\infty} x^{[-\lambda]_+} y(x) = 0, \tag{1.5}$$

where

$$[-\lambda]_+ = \begin{cases} -\lambda, & \lambda < 0, \\ 0, & \lambda \geq 0. \end{cases}$$

In recent years, the equation (1.4) with $\alpha = 0, m = 1$ has also caused much of attention [1, 2, 5].

Definition A function $y(x)$ is said to be a solution of the problem (1.4)–(1.5), if the following conditions are satisfied:

1) $y(x) \in C^1[0, +\infty)$,

2) the function $x^{\alpha+n-1}|(|y|^{m-1}y)'|^{N-1}(|y|^{m-1}y)'$ is continuously differentiable in $[0, +\infty)$, and (1.4) a.e. holds in $[0, +\infty)$.

3) (1.5) holds.

2. The two-point boundary value problem

In this section, we first study the properties of the solution for the problem (1.4)-(1.5).

Proposition 2.1. *Suppose the function $y(x) \in C^1[0, +\infty)$ is a solution of the problem* (1.4)-(1.5). *Then $y(x) \geq 0$, $y'(x) \leq 0$, as $\lambda \geq 0$.*

Proof. Otherwise, if there exists a point $x^* \in (0, +\infty)$ such that $y(x^*) < 0$. By $y(0) = 1$ and $y(+\infty) = 0$, we know that there would exist a point $a_1 \in (0, +\infty)$ and $y(a_1)$ is a minimum value. Hence, we have $y'(a_1) = 0$. Again by $y(+\infty) = 0$, $y(a_1) < 0$, we know that there exists a point $a_2 \in [a_1, +\infty)$, such that $y(a_2) = \frac{1}{2}y(a_1)$. In addition, the mean value theorem yields,

$$y(a_2) - y(a_1) = y'(b)(a_2 - a_1),$$

where $b \in [a_1, a_2]$. Since $y(a_2) - y(a_1) > 0$ and $a_2 - a_1 > 0$, we obtain $y'(b) > 0$. Taking $[c_1, c_2] \subset [a_1, a_2]$ such that $y'(c_1) = 0$ and $y(x) < 0$, $y'(x) > 0$ in (c_1, c_2). Integrating the equation (1.4) with respect to x over (c_1, c_2), we see that

$$\begin{aligned}
&\int_{c_1}^{c_2} \left[x^{\alpha+n-1} \left|(|y|^{m-1}y)'\right|^{N-1} (|y|^{m-1}y)'\right]' dx \\
&\qquad = c_2^{\alpha+n-1} \left|(|y|^{m-1}y)'\right|^{N-1} (|y|^{m-1}y)'(c_2) \\
&\qquad - c_1^{\alpha+n-1} \left|(|y|^{m-1}y)'\right|^{N-1} (|y|^{m-1}y)'(c_1) \\
&\qquad\qquad = c_2^{\alpha+n-1} \left|(|y|^{m-1}y)'\right|^{N-1} (|y|^{m-1}y)'(c_2) > 0.
\end{aligned}$$

Observing that $\lambda x^{n-1}y(x) \leq 0$ and $x^n y'(x) > 0$, we have $\lambda x^{n-1}y(x) - x^n y'(x) < 0$. Hence

$$\int_{c_1}^{c_2} \lambda x^{n-1}y(x) - x^n y'(x) dx \leq 0.$$

The contradiction shows $y \geq 0$.

Similarly, we can prove $y'(x) \leq 0$. Otherwise, if there exists $x^* \in (0, +\infty)$ such that $y'(x^*) > 0$. By $y(0) = 1$, $y(+\infty) = 0$, we know that there would exist a $x_1 \in (0, +\infty)$ with $y(x_1)$ being a maximum value. Hence, we have $y'(x_1) = 0$. By $y(x_1) > 0$ which is a maximum value, there exists a point $x_2 \in [x_1, +\infty)$, satisfying $y(x_2) = \frac{1}{2}y(x_1)$. The mean value theorem yields,

$$y(x_2) - y(x_1) = y'(b)(x_2 - x_1),$$

where $b \in [x_1, x_2]$. By $y(x_2) - y(x_1) < 0$ and $x_2 - x_1 > 0$, therefore we have $y'(b) < 0$. we take $[c_1, c_2] \subset [a_1, a_2]$ such that $y'(c_1) = 0$ and $y(x) > 0$, $y'(x) < 0$ in (c_1, c_2). Integrating the equation (1.4) with respect to x over (c_1, c_2), we see that

$$\begin{aligned}\int_{c_1}^{c_2} &\left[x^{\alpha+n-1} \left|(|y|^{m-1}y)'\right|^{N-1} (|y|^{m-1}y)'\right]' dx \\ &= c_2^{\alpha+n-1} \left|(|y|^{m-1}y)'\right|^{N-1} (|y|^{m-1}y)'(c_2) \\ &- c_1^{\alpha+n-1} \left|(|y|^{m-1}y)'\right|^{N-1} (|y|^{m-1}y)'(c_1) \\ &= c_2^{\alpha+n-1} \left|(|y|^{m-1}y)'\right|^{N-1} (|y|^{m-1}y)'(c_2) < 0.\end{aligned}$$

Observing that $\lambda x^{n-1}y(x) > 0$ and $-x^n y'(x) > 0$, then $\lambda x^{n-1}y(x) - x^n y'(x) > 0$. Thus we have

$$\int_{c_1}^{c_2} \lambda x^{n-1}y(x) - x^n y'(x)dx \geq 0.$$

The contradiction shows $y' \leq 0$. □

Proposition 2.2. *If the function $y(x) \in C^1[0, +\infty)$ is a solution of the problem* (1.4)-(1.5)*. Then $y(x) \geq 0$, $y'(x) \leq 0$ as $-n \leq \lambda < 0$.*

To prove the proposition 2.2, we need some lemmas. We first have

Lemma 2.1. *If there exists a x_0 satisfies $y'(x_0) = 0$, $y(x_0) > 0$. Then, we have $y(x) \geq 0$, $y'(x) \leq 0$ in $[x_0, +\infty)$, for $-n \leq \lambda < 0$.*

Proof. We first prove $y(x) \geq 0$. Otherwise, if there exists a point a_1 such that $y(a_1) < 0$. By $y(+\infty) = 0$, we know that there would exist a point x_1 and $y(x_1) < 0$ is a minimum value. Hence, we have $y'(x_1) = 0$. Integrating the equation (1.4) with respect to x over (x_1, x_2), and using $y'(x_0) = 0$, $y'(x_1) = 0$ and $y(x_0) > 0$, $y(x_1) < 0$, we see that

$$\begin{aligned}\int_{x_0}^{x_1} &\left[x^{\alpha+n-1} \left|(|y|^{m-1}y)'\right|^{N-1} (|y|^{m-1}y)'\right]' dx \\ &= x_1^{\alpha+n-1} \left|(|y|^{m-1}y)'\right|^{N-1} (|y|^{m-1}y)'(x_1) \\ &\quad - x_0^{\alpha+n-1} \left|(|y|^{m-1}y)'\right|^{N-1} (|y|^{m-1}y)'(x_0) = 0,\end{aligned}$$

and

$$\begin{aligned}
&\int_{x_0}^{x_1} \lambda x^{n-1}y(x) - x^n y'(x)dx \\
&\qquad = \int_{x_0}^{x_1} \lambda x^{n-1}y(x)dx + \int_{x_0}^{x_1} nx^{n-1}y(x)dx - x^n y\Big|_{x_0}^{x_1} \\
&\qquad > (\lambda+n)\frac{1}{n}y(x_1)(x_1^n - x_0^n) + x_0^n y(x_0) - x_1^n y(x_1) \\
&\qquad = (\frac{\lambda}{n}+1)y(x_1)x_1^n - (\frac{\lambda}{n}+1)y(x_1)x_0^n + x_0^n y(x_0) - x_1^n y(x_1) \\
&\qquad = \frac{\lambda}{n}y(x_1)x_1^n - (\frac{\lambda}{n}+1)y(x_1)x_0^n + x_0^n y(x_0) > 0.
\end{aligned}$$

The contradiction shows $y \geq 0$. Similarly, we can prove $y'(x) \leq 0$. □

Lemma 2.2. *If there exists a x_0 satisfies $y'(x_0) = 0$, $y(x_0) < 0$, then, we have $y(x) \leq 0$, $y'(x) \geq 0$ in $[x_0, +\infty)$, for $-n \leq \lambda < 0$.*

Proof. For simplicity, we only prove the first inequality $y(x) \leq 0$, since the other can be shown similarly. Otherwise, if there exists a_1 such that $y(a_1) > 0$, by $y(+\infty) = 0$, we know that there would exist a x_1 and $y(x_1) < 0$ is a maximum value. Hence, we have $y'(x_1) = 0$. By $y'(x_0) = 0$, $y'(x_1) = 0$ and $y(x_0) > 0$, $y(x_1) < 0$. Integrating the equation (1.4) with respect to x over (x_1, x_2), we see that

$$\begin{aligned}
&\int_{x_0}^{x_1} \left[x^{\alpha+n-1}\left|(|y|^{m-1}y)'\right|^{N-1}(|y|^{m-1}y)'\right]' dx \\
&\qquad = x_1^{\alpha+n-1}\left|(|y|^{m-1}y)'\right|^{N-1}(|y|^{m-1}y)'(x_1) \\
&\qquad\qquad - x_0^{\alpha+n-1}\left|(|y|^{m-1}y)'\right|^{N-1}(|y|^{m-1}y)'(x_0) = 0
\end{aligned}$$

and

$$\begin{aligned}
&\int_{x_0}^{x_1} \lambda x^{n-1}y(x) - x^n y'(x)dx \\
&\qquad = \int_{x_0}^{x_1} \lambda x^{n-1}y(x)dx + \int_{x_0}^{x_1} nx^{n-1}y(x)dx - x^n y\Big|_{x_0}^{x_1} \\
&\qquad < (\lambda+n)\frac{1}{n}y(x_1)(x_1^n - x_0^n) + x_0^n y(x_0) - x_1^n y(x_1) \\
&\qquad = (\frac{\lambda}{n}+1)y(x_1)x_1^n - (\frac{\lambda}{n}+1)y(x_1)x_0^n + x_0^n y(x_0) - x_1^n y(x_1) \\
&\qquad = \frac{\lambda}{n}y(x_1)x_1^n - (\frac{\lambda}{n}+1)y(x_1)x_0^n + x_0^n y(x_0) < 0
\end{aligned}$$

The contradiction shows $y \leq 0$. □

Proof of Proposition 2.2. We first prove that $y(x) \geq 0$. Otherwise, if there exists $x^* \in (0, +\infty)$ such that $y(x^*) < 0$, by $y(0) = 1$, $y(+\infty) = 0$, we know that there would exist a $a_1 \in (0, +\infty)$ and $y(a_1)$ is a minimum value. Therefore, we have $y'(a_1) = 0$. By lemma 2.2, we have $y'(x) \geq 0$, $y(x) \leq 0$ in $[a_1, +\infty)$. Multiplying both sides of the equation (1.4) by $-x^{-\lambda-1}$, we have

$$-x^{-\lambda-n}\left[x^{\alpha+n-1}\left|(|y|^{m-1}y)'\right|^{N-1}(|y|^{m-1}y)'\right]' = [x^{-\lambda}y(x)]' \tag{2.1}$$

Then, integrating the resulting relation with respect to x over (a_1, M), we have

$$\begin{aligned}\int_{a_1}^{M} & -x^{-\lambda-n}\left[x^{\alpha+n-1}\left|(|y|^{m-1}y)'\right|^{N-1}(|y|^{m-1}y)'\right]' dx \\ &= -x^{-\lambda-n}x^{\alpha+n-1}\left|(|y|^{m-1}y)'\right|^{N-1}(|y|^{m-1}y)'\Big|_{x_2}^{M} \\ &- (\lambda+n)\int_{a_1}^{M} x^{\alpha+n-1}\left|(|y|^{m-1}y)'\right|^{N-1}(|y|^{m-1}y)'x^{-\lambda-n-1}dx \\ &= M^{-\lambda+\alpha-1}\left|(|y|^{m-1}y)'\right|^{N-1}(|y|^{m-1}y)'(M) \\ &- (\lambda+n)\int_{a_1}^{M} x^{\alpha+n-1}\left|(|y|^{m-1}y)'\right|^{N-1}(|y|^{m-1}y)'x^{-\lambda-n-1}dx \leq 0\end{aligned}$$

However, the right hand side

$$\int_{a_1}^{M}(x^{-\lambda}y)'dx = M^{-\lambda}y(M) - a_1^{-\lambda}y(a_1).$$

By $\lim x^{-\lambda}y(x) = 0$, we have the right hand side > 0, as $M \to +\infty$. The contradiction shows $y \leq 0$.

Similarly, we can prove $y'(x) \geq 0$. □

Proposition 2.3. *Suppose the function $y(x) \in C^1[0, +\infty)$ is a solution of the problem* (1.4)-(1.5). *Then a.e. $y'(x) < 0$, as $y(x) > 0$, where $\lambda \neq 0$.*

Proof. If there exists a point $x_0 \in [0, +\infty)$, such that $y(x_0) > 0$ and $y'(x_0) = 0$. Without loss of generality, we assume there exists a strictly monotone sequence $\{x_j\}_{j=1}^{\infty}$ such that $\lim_{j\to+\infty} x_j = x_0$, $y(x_j) > 0$, $y'(x_j) = 0$. Set $f(x_j) = x_j^{\alpha+n-1}\left|(|y|^{m-1}y(x_j))'\right|^{N-1}(|y|^{m-1}y(x_j))'$. Hence

$$\begin{aligned}0 = \frac{f(x_{j+1}) - f(x_j)}{x_{j+1} - x_j} &= \left[x^{\alpha+n-1}\left|(|y|^{m-1}y)'\right|^{N-1}(|y|^{m-1}y)'\right]'\Big|_{x=\xi_j} \\ &= \lambda {\xi_j}^{n-1}y(\xi_j) - {\xi_j}^{n}y'(\xi_j),\end{aligned}$$

where $\xi_j \in [x_j, x_{j+1}]$. Letting $j \to +\infty$, we have $y(x_0) = 0$. The contradiction shows $y' < 0$. $\square$

3. Self-similar solution

To prove the existence of solutions of the problem (1.4)-(1.5), we set

$$v(t) = mt^{m-1}W^{\alpha+n-1}|W'|^{-N},$$

and consider the following problem

$$\frac{dv}{dt} = -\lambda t W^{n-1}W' + W^n, \tag{3.1}$$

$$\frac{dW^{(N-n+\alpha+1)/N}}{dt} = -\frac{N-n+\alpha+1}{N}m^{1/N}t^{(m-1)/N}v^{-1/N}(t), \tag{3.2}$$

$$v(0) = 0, \quad W(1) = 0. \tag{3.3}$$

To prove the existence of solution of the problem (3.1)-(3.3), we consider the problem (3.1), (3.2) and

$$v(0) = h > 0, \quad W(1) = 0. \tag{3.4}$$

Theorem 3.1. *For fixed $h > 0$, the problem* (3.1), (3.2), (3.4) *admits a solution $v(t, h)$.*

Proof. If $v(t) \in C[0, 1]$ satisfies

$$\begin{aligned} v(t) = {} & \lambda m^{1/N}\int_0^t \tau^{(N+m-1)/N}v^{-1/N}(\tau) \\ & \left(\frac{N-n+\alpha+1}{N}m^{1/N}\int_\tau^1 s^{(m-1)/N}v^{-1/N}(s)ds\right)^{(Nn-N+\alpha+n-1)/(N-1)} d\tau \\ & + \int_0^t \left(\frac{N-n+\alpha+1}{N}m^{1/N}\int_\tau^1 s^{(m-1)/N}v^{-1/N}(s)ds\right)^{Nn/(N-1)} d\tau + h \end{aligned} \tag{3.5}$$

and let

$$W(t) = \left(\frac{N-n+\alpha+1}{N}m^{1/N}\int_t^1 s^{(m-1)/N}v^{-1/N}(s)ds\right)^{N/(N-1)}. \tag{3.6}$$

It is seen that $(v(t), W(t))$ is a solution of the problem (3.1), (3.2), (3.4).

On the other hand, clearly, for any $a \in [0,1]$, we have

$$v(t) = \lambda m^{1/N} \int_a^t \tau^{(N+m-1)/N} v^{-1/N}(\tau) \left(\frac{N-n+\alpha+1}{N} m^{1/N} \int_\tau^1 s^{(m-1)/N} v^{-1/N}(s) ds\right)^{(Nn-N+\alpha+n-1)/(N-1)} d\tau + \int_a^t \left(\frac{N-n+\alpha+1}{N} m^{1/N} \int_\tau^1 s^{(m-1)/N} v^{-1/N}(s) ds\right)^{Nn/(N-1)} d\tau + v(a). \tag{3.7}$$

Define the map $\varphi : \Omega \to \Omega$,

$$\varphi v = \lambda m^{1/N} \int_0^t \tau^{(N+m-1)/N} v^{-1/N}(\tau) \left(\frac{N-n+\alpha+1}{N} m^{1/N} \int_\tau^1 s^{(m-1)/N} v^{-1/N}(s) ds\right)^{(Nn-N+\alpha+n-1)/(N-1)} d\tau + \int_0^t \left(\frac{N-n+\alpha+1}{N} m^{1/N} \int_\tau^1 s^{(m-1)/N} v^{-1/N}(s) ds\right)^{Nn/(N-1)} d\tau + h,$$

where $\Omega = \{v(t) \in C[0,1]; h \le v(t) \le (\varphi h)(t)\}$. It is seen that the operator φ is Ω to Ω continuous and compact. By Leray–Schauder principle of fixed point, the operator φ has a fixed point $v(t,h)$ in Ω, which is the desired solution of the (3.5). Hence it is the solutions of the problem (3.1), (3.2), (3.4). $\square$

Lemma 3.1. *If $h_1 > h_2 > 0$, then*

$$0 \le v(t,h_1) - v(t,h_2) \le h_1 - h_2, \quad on\ [0,1].$$

Proof. We first show the left inequality. If this were not true, then there will be a point $t_0 \in [0,1]$, such that

$$v(t_0,h_1) - v(t_0,h_2) < 0.$$

By $v(0,h_1) - v(0,h_2) = h_1 - h_2 > 0$, hence $t_0 \ne 0$.

Since $t_0 \ne 0$, then there exists a interval $(a,t_0]$, where $0 < a < t_0 \le 1$, such that

$$v(t,h_1) - v(t,h_2) < 0, \quad v(a,h_1) - v(a,h_2) = 0 \ \text{ in}(a,t_0].$$

Using (3.7), we obtain $v(t_0,h_1) - v(t_0,h_2) = 0$, this yields a contradiction.

We can obtain the right inequality, by the left inequality and (3.5). $\square$

By the lemma 3.1, we have the following conclusion.

Lemma 3.2. *The problem* (3.1), (3.2), (3.4) *has a unique solution.*

Theorem 3.2. *The problem* (3.1), (3.2), (3.3) *has a unique solution* $v(t, h)$.

Proof. By the lemma 3.1, we know that

$$\lim_{h\to 0} v(t,h) = v(t) \text{ uniformly for } t \in [0,1].$$

Substituting $v(t,h)$ into (3.5), then letting $h \to 0$, using lemma 3.2, we have the problem (3.1)-(3.3) admits one and only one nonnegative continuous solution $v(t)$. □

Theorem 3.3. *The problem* (1.4), (1.5) *has a unique solution.*

Proof. Now, we construct a solution of the problem (1.4)-(1.5) by the solution of the problem (3.1)-(3.3). Suppose $(v(t), W(t))$ is a solution of the problem (3.1)-(3.3), then $W(t)$ is a strictly decreasing function in $[0,1]$. Hence the inverse function $t = y(x)$ of $x = W(t)$ exists in $[0, W(0))$. If $W(0) < +\infty$, we define the $y(x) = 0$, as $x \in [W(0), +\infty)$. Hence the $y(x)$ is a nonnegative function with $y(0) = 1$, $\lim\limits_{x\to+\infty} x^{[-\lambda]_+} y(x) = 0$, in $[0, +\infty)$. We set $x_0 = W(0)$. Observing that $x = W(t)$ in $[0, x_0)$ and $y'(x) = \frac{1}{W'(t)} < 0$ a.e. in $(0, x_0)$. As $x_0 < +\infty$, we have $y'_-(x_0) = \frac{1}{W'_+(0)} = 0$. Again as $x \in [x_0, +\infty)$, $y(x) = 0$, hence, we know that $y'(x)$ continuous at x_0 and $y'(x_0) = 0$. As $x_0 = +\infty$, by (3.6), we have $y'(x) = -v^{1/N}(t)/[(N - n + \alpha - 1)/(N - 1)W^{1/N}]$. Again by $v(0) = 0$ and $W(0) = +\infty$, we know that $y'(+\infty) = 0$. Substituting $t = y(x)$ in (1.4), (1.5), it is easily seen that the function $y(x) \in C^1[0, +\infty)$ is the solution of the problem (1.4), (1.5). □

By the Theorem 3.3, we have the following conclusion.

Theorem 3.4. *If* $m > 1$, $N > 1$, $\gamma > 0$, $u_0 > 0$, $\frac{\gamma}{\beta} \geq -n$. *Then the problem* (1.1)–(1.3) *admits one and only one nonnegative self-similar solution*

$$u(r,t) = u_0 t^\alpha y(crt^{-\beta}),$$

where $y(x) \in C^1[0, +\infty)$ *is a unique nonnegative solution of the problem* (1.4), (1.5), c, β *are defined in Section* 1.

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