Abstract. The main results are summarized by Figure 1. They demonstrate the resiliency of the isomorphism constructed in [Nai99] between weighted cohomology and a variant of weighted L_2 cohomology. Our attention is restricted from generic locally symmetric spaces to spaces whose ends are hyperbolic, diffeomorphic to $(0,\infty)\times(S^1)^{n-1}$, and carry exponentially warped product metrics. For weighting functions which are exponential in the Busemann coordinates of these ends, the standard w weighted L_2 cohomology will be utilized in lieu of the variant defined in [Fra98]. The resulting standard w weighted L_2 cohomology groups may be infinite dimensional vector spaces, but the precise weighting functions at which this undesirable behavior occurs are characterized. For the remaining exponential weights, the w weighted L_2 cohomology is again an analogue of weighted cohomology. An immediate consequence of finite dimensionality of the standard w weighted L_2 groups is a w weighted Hodge theory summarized by a strong w weighted Kodaira decomposition. This is outlined in the introduction.

After the asymptotically hyperbolic case is complete, the literature on weighted Hardy inequalities on the half line is used to derive certain extensions to some non-hyperbolic end metrics and non-exponential weighting functions. The two most immediate applications are as follows. First, say a function on the half line $k(t)$ satisfies $k'k^{-1} \leq -c$ for $c > 0$. Then one may replace the exponential in the metric of $(0,\infty)\times(S^1)^{n-1}$ by $k(t)$ and weight by powers of $k(t)$ rather than e^{-t}, and Figure 1 holds. Second, the analysis allows one to consider weighting functions which on each end are $w(t) = \mathrm{e}^{\alpha t^2}$ for $\alpha \in \mathbb{R}$. These weighting functions compute either de Rham cohomology or compactly supported de Rham cohomology when $\alpha < 0$ or $\alpha > 0$, respectively.