ABSTRACT. The $\bar{\partial}_J$ operator over an almost complex manifold induces canonical connections of type $(0,1)$ over the bundles of $(p,0)$-forms. If the almost complex structure is integrable then the previous connections induce the canonical holomorphic structures of the bundles of $(p,0)$-forms. For $p=1$ we can extend the corresponding connection to all Schur powers of the bundle of $(1,0)$-forms. Moreover using the canonical $\mathbb{C}$-linear isomorphism betwen the bundle of $(1,0)$-forms and the complex cotangent bundle $T^*_{X,J}$ we deduce canonical connections of type $(0,1)$ over the Schur powers of the complex cotangent bundle $T^*_{X,J}$. If the almost complex structure is integrable then the previous $(0,1)$-connections induce the canonical holomorphic structures of those bundles. In the nonintegrable case those $(0,1)$-connections induce just the holomorphic canonical structures of the restrictions of the corresponding bundles to the images of smooth J-holomorphic curves. We introduce the notion of Chern curvature for those bundles. The geometrical meaning of this notion is a natural generalisation of the classical notion of Chern curvature for the holomophic vector bundles over a complex manifold. We have a particular interest for the case of the tangent bundle in view of applications concerning the regularisation of J-plurisubharmonic functions by means of the geodesic flow induced by a Chern connection on the tangent bundle. This method has been used by Demailly, 1994, in the complex integrable case. Our specific study in the case of the tangent bundle gives an asymptotic expanson of the Chern flow which relates in an optimal way the geometric obstructions caused by the torsion of the almost complex structure, and the nonsymplectic nature of the metric.