

ABSTRACT. Let  $\Gamma$  be a torsion-free cocompact lattice in  $\text{Aut}(\mathcal{T}_1) \times \text{Aut}(\mathcal{T}_2)$ , where  $\mathcal{T}_1, \mathcal{T}_2$  are trees whose vertices all have degree at least three. The group  $H_2(\Gamma, \mathbb{Z})$  is determined explicitly in terms of an associated 2-dimensional tiling system. It follows that under appropriate conditions the crossed product  $C^*$ -algebra  $\mathcal{A}$  associated with the action of  $\Gamma$  on the boundary of  $\mathcal{T}_1 \times \mathcal{T}_2$  satisfies  $\text{rank } K_0(\mathcal{A}) = 2 \cdot \text{rank } H_2(\Gamma, \mathbb{Z})$ .