

ABSTRACT. Let  $E/k$  be an elliptic curve over a number field. We obtain some quantitative refinements of results of Hindry–Silverman, giving an upper bound for the number of  $k$ -rational torsion points, and a lower bound for the canonical height of non-torsion  $k$ -rational points, in terms of expressions depending explicitly on the degree  $d = [k : \mathbb{Q}]$  of  $k$  and the Szpiro ratio  $\sigma$  of  $E/k$ . The bounds exhibit only polynomial dependence on both  $d$  and  $\sigma$ .