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Corrigendum to "Topology and arithmetic of resultants, I", New York J. Math. 22 (2016), 801–821

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Abstract. This note is meant to correct a mistake in [1]. A corrected version of [1] can be found on the archive: arXiv:1506.02713.

In Step 2 of Theorem 1.2 on page 808 of [1], we claimed that the map of Equation (3.3) (the map Ψ in Equation (1) below) is an isomorphism. This is not true, as pointed out to us by H. Spink and D. Tseng. However, we will see below that it is a bijective morphism. This has the effect that one needs to add the assumption that $\mathrm{char}(K) = 0$ in Theorem 1.2, Corollary 1.3, and Theorem 1.7 of [1]. The corresponding point counts over $\mathbb{F}_q$ still hold.

Step 2 of Theorem 1.2. As to the proof of Theorem 1.2 on page 808 of [1], the entirety of Step 2 should be deleted and replaced by the following.

Let $k \geq 0$. Define a morphism

$$\overline{\Psi} : \mathbb{A}^{m(d-nk)} \times \mathbb{A}^k \to \mathbb{A}^{md}$$

by

$$\overline{\Psi}(f_1, \ldots, f_m, g) := (f_1 g^n, \ldots, f_m g^n).$$

The restriction of $\overline{\Psi}$ to $Poly_n^{d-kn,m} \times \mathbb{A}^k$ gives a morphism

$$\Psi : Poly_n^{d-kn,m} \times \mathbb{A}^k \to R_{n,k}^{d,m} - R_{n,k+1}^{d,m} \tag{1}$$

where the target is the space of m-tuples of degree d polynomials with a common n-fold factor of degree equal to k, with no other common n-fold factors. We think of the map Ψ^{-1} as the (non-algebraic) map that extracts a common n-fold factor from a tuple of polynomials. We claim that:

(i) For any field k the morphism Ψ is bijective.
(ii) For $k = \mathbb{C}$, the map Ψ is a homeomorphism in the classical topology.

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These facts will allow us to analyze $Poly_n^{d,m}$ recursively. Note that the case $k = 0$ follows by definition:

$$Poly_n^{d,m} := R_{n,0}^{d,m} - R_{n,1}^{d,m}.$$

To see (i): It is clear from the definitions that Ψ is surjective. The map Ψ is injective because there is a unique n-fold degree k factor in each $f_i g^n$, so if $f_i g^n = u_i v^n$ then this implies $g = v$ and so $f_i = u_i$.

To see (ii): First note that the spaces of polynomials in the range and domain of $\overline{\Psi}$ have Galois covers given by the corresponding spaces of (all possible orderings of) roots, with deck group the appropriate product of symmetric groups. The map $\overline{\Psi}$ lifts to a map between these spaces of roots:

$$\Phi : \mathbb{A}^{m(d-nk)} \times \mathbb{A}^k \to \mathbb{A}^{md}$$

given by

$$\Phi((\vec{r}_1, \ldots, \vec{r}_m), \vec{s})) := ((\vec{r}_1, (\vec{s})^n), \ldots, (\vec{r}_m, (\vec{s})^n))$$

where $\vec{r}_i$ is the vector of d roots of f_i; the vector of roots of g is denoted $\vec{s}$; and where $(\vec{s})^n$ denotes the vector $(\vec{s}, \ldots, \vec{s})$, where $\vec{s}$ is repeated n times. It follows that the map Φ is closed, and hence the map $\overline{\Psi}$ is closed, and hence the map Ψ is closed. Since Ψ is bijective, it follows that Ψ is a homeomorphism.

Step 3 of Theorem 1.2. In Step 3 on page 808, one should insert the following after Equation (3.6).

We now claim that, when $\mathrm{char}(K) = 0$ then

$$[Poly_n^{d-kn,m}] \cdot \mathbb{L}^k = [R_{n,k}^{d,m}] - [R_{n,k+1}^{d,m}] \tag{2}$$

To see this, first note that we proved in Step 2 that the map Ψ in (1) is a bijective morphism on K-points for all fields K. It is known (see, e.g., Remark 4.1 of [2]) that if $\mathrm{char}(K) = 0$ then a bijective morphism of K-varieties induces an equality $[X] = [Y]$ in the Grothendieck ring of K-varieties.

The line "Plugging in the expression from Equation (3.3) into Equation (3.6)" should now read: "Plugging in the expression from Equation (2) into Equation 3.6 "

References

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