Abstract. We establish the existence of three new subgroups of the group of volume-preserving diffeomorphisms of a compact n-dimensional ($n \geq 2$) Riemannian manifold which are associated with the Dirichlet, Neumann, and Mixed type boundary conditions that arise in second-order elliptic PDEs. We prove that when endowed with the H^s Hilbert-class topologies for $s > (n/2)+1$, these subgroups are C^∞ differential manifolds. We consider these new diffeomorphism groups with an H^1-equivalent right invariant metric, and prove the existence of unique smooth geodesics $\eta(t,\cdot)$ of this metric, as well as existence and uniqueness of the Jacobi equations associated to this metric. Geodesics on these subgroups are, in fact, the flows of a time-dependent velocity vector field $u(t,x)$, so that $\partial_t \eta(t,\cdot) = u(t,\eta(t,\cdot))$ with $\eta(0,x) = x$, and remarkably the vector field $u(t,x)$ solves the so-called Lagrangian averaged Euler (LAE-α) equations on M. These equations, and their viscous counterparts, the Lagrangian averaged Navier-Stokes (LANS-α) equations, model the motion of a fluid at scales larger than an a priori fixed parameter $\alpha > 0$, while averaging (or filtering-out) the small scale motion, and this is achieved without the use of artificial viscosity. We prove that for divergence-free initial data satisfying $u = 0$ on ∂M, the LAE-α equations are well-posed, globally when $n = 2$. We also find the boundary conditions that make the LANS-α equations well-posed, globally when $n = 3$, and prove that solutions of the LANS-α equations converge when $n = 2,3$ for almost all t in some fixed time interval $(0,T)$ in H^s, $s \in (n/2+1, 3)$ to solutions of the LAE-α equations, thus confirming the scaling arguments of Barenblatt & Chorin.