Abstract. We show that under suitable conditions a branched cover satisfies the same upper curvature bounds as its base space. First we do this when the base space is a metric space satisfying Alexandrov's curvature condition CAT(κ) and the branch locus is complete and convex. Then we treat branched covers of a Riemannian manifold over suitable mutually orthogonal submanifolds. In neither setting do we require that the branching be locally finite. We apply our results to hyperplane complements in several Hermitian symmetric spaces of nonpositive sectional curvature in order to prove that two moduli spaces arising in algebraic geometry are aspherical. These are the moduli spaces of the smooth cubic surfaces in $\mathbb{C}P^3$ and of the smooth complex Enriques surfaces.