ABSTRACT. We identify a distinguished hyperbolic partial differential equation of importance in the study of principal foliations. We apply Riemann's method to obtain an obstruction to the occurrence of elliptic sectors in principal foliations on surfaces and as a consequence we obtain the first global result on the Carathéodory conjecture.