

ABSTRACT. We construct smooth closed hypersurfaces of positive curvature with prescribed submanifolds and tangent planes. Further, we develop some applications to boundary value problems via Monge-Ampère equations, smoothing of convex polytopes, and an extension of Hadamard's ovaloid theorem to hypersurfaces with boundary.