ABSTRACT. A theorem of E. Lerman and S. Tolman, generalizing a result of T. Delzant, states that compact symplectic toric orbifolds are classified by their moment polytopes, together with a positive integer label attached to each of their facets. In this paper we use this result, and the existence of "global" action-angle coordinates, to give an effective parametrization of all compatible toric complex structures on a compact symplectic toric orbifold, by means of smooth functions on the corresponding moment polytope. This is equivalent to parametrizing all toric Kähler metrics and generalizes an analogous result for toric manifolds.

A simple explicit description of interesting families of extremal Kähler metrics, arising from recent work of R. Bryant, is given as an application of the approach in this paper. The fact that in dimension four these metrics are self-dual and conformally Einstein is also discussed. This gives rise in particular to a one parameter family of self-dual Einstein metrics connecting the well known Eguchi-Hanson and Taub-NUT metrics.