ABSTRACT. Using the Gauss-Manin connection (Picard-Fuchs differential equation) and a result of Malgrange, a special class of algebraic solutions to isomonodromic deformation equations, the *geometric isomonodromic deformations*, is defined from "families of families" of algebraic varieties. Geometric isomonodromic deformations arise naturally from combinatorial strata in the moduli spaces of elliptic surfaces over \mathbb{P}^1 . The complete list of geometric solutions to the Painlevé VI equation arising in this way is determined. Motivated by this construction, we define another class of algebraic isomonodromic deformations whose monodromy preserving families arise by "pullback" from (rigid) local systems. Using explicit methods from the theory of Hurwitz spaces, all such algebraic Painlevé VI solutions coming from arithmetic triangle groups are classified.