Abstract. Let M be an asymptotically flat 3-manifold of nonnegative scalar curvature. The Riemannian Penrose Inequality states that the area of an outermost minimal surface N in M is bounded by the ADM mass m according to the formula $|N| \leq 16\pi m^2$. We develop a theory of weak solutions of the inverse mean curvature flow, and employ it to prove this inequality for each connected component of N using Geroch's monotonicity formula for the ADM mass. Our method also proves positivity of Bartnik's gravitational capacity by computing a positive lower bound for the mass purely in terms of local geometry.