ABSTRACT. Closed hyperbolic manifolds are proven to minimize volume over all Alexandrov spaces with curvature bounded below by -1 in the same bilipschitz class. As a corollary compact convex cores with totally geodesic boundary are proven to minimize volume over all hyperbolic manifolds in the same bilipschitz class. Also, closed hyperbolic manifolds minimize volume over all hyperbolic cone-manifolds in the same bilipschitz class with cone angles $< 2\pi$. The proof uses techniques developed by Besson-Courtois-Gallot. In 3 dimensions, this result provides a partial solution to a conjecture in Kleinian groups concerning acvlindrical manifolds.