

ABSTRACT. We prove that the \widehat{A} -genus vanishes on certain non-spin manifolds. Namely, $\widehat{A}(M)$ vanishes on any oriented, compact, connected, smooth manifold M with finite second homotopy group and endowed with non-trivial (isometric) smooth S^1 actions. This result extends that of Atiyah and Hirzebruch on spin manifolds endowed with smooth S^1 actions [1] to manifolds which are not necessarily spin.

We prove such vanishing by means of the elliptic genus defined by Ochanine [23,24], showing that it also has the special property of being “rigid under S^1 actions” on these (not necessarily spin) manifolds.

We conclude with a non-trivial application of this new vanishing theorem by classifying the positive quaternion-Kähler 12-manifolds. Namely, we prove that every quaternion-Kähler 12-manifold with a complete metric of positive scalar curvature must be a symmetric space.