Abstract. We study the behavior of the Cheeger isoperimetric constant on infinite families of graphs and Riemann surfaces, and its relationship to the first eigenvalue λ_1 of the Laplacian. We adapt probabilistic arguments of Bollobás to the setting of Riemann surfaces, and then show that Cheeger constants of the modular surfaces are uniformly bounded from above away from the maximum value. We extend this result to the class of Ramanujan surfaces, defined below.