Abstract. We show that there is a C^∞ open and dense set of positively curved metrics on S^2 whose geodesic flow has positive topological entropy, and thus exhibits chaotic behavior. The geodesic flow for each of these metrics possesses a horseshoe and it follows that these metrics have an exponential growth rate of hyperbolic closed geodesics. The positive curvature hypothesis is required to ensure the existence of a global surface of section for the geodesic flow. Our proof uses a new and general topological criterion for a surface diffeomorphism to exhibit chaotic behavior.

Very shortly after this manuscript was completed, the authors learned about remarkable recent work by Hofer, Wysocki, and Zehnder [14,15] on three dimensional Reeb flows. In the special case of geodesic flows on S^2, they show that if the geodesic flow has no parabolic closed geodesics (this holds for an open and C^∞ dense set of Riemannian metrics on S^2), then it possesses either a global surface of section or a heteroclinic orbit. It then immediately follows from the proof of our main theorem that there is a C^∞ open and dense set of Riemannian metrics on S^2 whose geodesic flow has positive topological entropy.

This concludes a program to show that every orientable compact surface has a C^∞ open and dense set of Riemannian metrics whose geodesic flow has positive topological entropy.