

ABSTRACT. The Yamabe invariant is an invariant of a closed smooth manifold defined using conformal geometry and the scalar curvature. Recently, Petean showed that the Yamabe invariant is non-negative for all closed simply connected manifolds of dimension ≥ 5 . We extend this to show that Yamabe invariant is nonnegative for all closed manifolds of dimension ≥ 5 with fundamental group of odd order having all Sylow subgroups abelian. The main new geometric input is a way of studying the Yamabe invariant on Toda brackets. A similar method of proof shows that all closed oriented manifolds of dimension ≥ 5 with non-spin universal cover, with finite fundamental group having all Sylow subgroups elementary abelian, admit metrics of positive scalar curvature, once one restricts to the “complement” of manifolds whose homology classes are “toral.” The exceptional toral homology classes only exist in dimensions not exceeding the “rank” of the fundamental group, so this proves important cases of the Gromov-Lawson-Rosenberg Conjecture once the dimension is sufficiently large.