ABSTRACT. Using the  $L^2$  norm of the Higgs field as a Morse function, we study the moduli spaces of U(p,q)-Higgs bundles over a Riemann surface. We require that the genus of the surface be at least two, but place no constraints on (p,q). A key step is the identification of the function's local minima as moduli spaces of holomorphic triples. In a companion paper [7] we prove that these moduli spaces of triples are nonempty and irreducible.

Because of the relation between flat bundles and fundamental group representations, we can interpret our conclusions as results about the number of connected components in the moduli space of semisimple PU(p, q)-representations. The topological invariants of the flat bundles are used to label subspaces. These invariants are bounded by a Milnor–Wood type inequality. For each allowed value of the invariants satisfying a certain coprimality condition, we prove that the corresponding subspace is nonempty and connected. If the coprimality condition does not hold, our results apply to the closure of the moduli space of irreducible representations.