

ABSTRACT. The loop space $L\mathbb{P}_1$ of the Riemann sphere consisting of all C^k or Sobolev $W^{k,p}$ maps $S^1 \rightarrow \mathbb{P}_1$ is an infinite dimensional complex manifold. The loop group $LPGL(2, \mathbb{C})$ acts on $L\mathbb{P}_1$. We prove that the group of $LPGL(2, \mathbb{C})$ -invariant holomorphic line bundles on $L\mathbb{P}_1$ is isomorphic to an infinite dimensional Lie group. Further, we prove that the space of holomorphic sections of any such line bundle is finite dimensional, and compute the dimension for a generic bundle.