ABSTRACT. 0-efficient triangulations of 3-manifolds are defined and studied. It is shown that any triangulation of a closed, orientable, irreducible 3-manifold $M$ can be modified to a 0-efficient triangulation or $M$ can be shown to be one of the manifolds $S^3$, $\mathbb{R}\mathrm{P}^3$ or $L(3,1)$. Similarly, any triangulation of a compact, orientable, irreducible, $\partial$-irreducible 3-manifold can be modified to a 0-efficient triangulation. The notion of a 0-efficient ideal triangulation is defined. It is shown if $M$ is a compact, orientable, irreducible, $\partial$-irreducible 3-manifold having no essential annuli and distinct from the 3-cell, then $\overset{\circ}{M}$ admits an ideal triangulation; furthermore, it is shown that any ideal triangulation of such a 3-manifold can be modified to a 0-efficient ideal triangulation. A 0-efficient triangulation of a closed manifold has only one vertex or the manifold is $S^3$ and the triangulation has precisely two vertices. 0-efficient triangulations of 3-manifolds with boundary, and distinct from the 3-cell, have all their vertices in the boundary and then just one vertex in each boundary component. As tools, we introduce the concepts of barrier surface and shrinking, as well as the notion of crushing a triangulation along a normal surface. A number of applications are given, including an algorithm to construct an irreducible decomposition of a closed, orientable 3-manifold, an algorithm to construct a maximal collection of pairwise disjoint, normal 2-spheres in a closed 3-manifold, an alternate algorithm for the 3-sphere recognition problem, results on edges of low valence in minimal triangulations of 3-manifolds, and a construction of irreducible knots in closed 3-manifolds.