

ABSTRACT. This paper extends to dimension 4 the results in the article “Second order families of special Lagrangian 3-folds” by Robert Bryant. We consider the problem of classifying the special Lagrangian 4-folds in \mathbb{C}^4 whose fundamental cubic at each point has a nontrivial stabilizer in $\mathrm{SO}(4)$. Points on special Lagrangian 4-folds where the $\mathrm{SO}(4)$ -stabilizer is nontrivial are the analogs of the umbilical points in the classical theory of surfaces. In proving existence for the families of special Lagrangian 4-folds, we used the method of exterior differential systems in Cartan-Kähler theory. This method is guaranteed to tell us whether there are any families of special Lagrangian submanifolds with a certain stabilizer type, but does not give us an explicit description of the submanifolds. To derive an explicit description, we looked at foliations by submanifolds and at other geometric particularities. In this manner, we settled many of the cases and described the families of special Lagrangian submanifolds in an explicit way.