ABSTRACT. We discuss the integration of Poisson brackets, motivated by our recent solution to the integrability problem for general Lie brackets. We give the precise obstructions to integrating Poisson manifolds, describing the integration as a symplectic quotient, in the spirit of the Poisson sigma-model of Cattaneo and Felder. For regular Poisson manifolds we express the obstructions in terms of variations of symplectic areas, improving on results of Alcalde Cuesta and Hector. We apply our results (and our point of view) to decide about the existence of complete symplectic realizations, to the integrability of submanifolds of Poisson manifolds, and to the study of dual pairs, Morita equivalence and reduction.