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COMMUTING MAPPINGS, FIXED POINTS AND ĆIRIĆ CONTRACTION IN UNIFORM SPACES

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Abstract. Some results on common fixed points for a pair of mappings defined on a sequentially complete Hausdororff uniform space have been obtained. Our work extends known results due to Ćirić and Jungck. Convergence theorems for sequences of fixed points are also established.

1. Introduction. Let (X, ρ) be a metric space. Clearly, a fixed point of a self-mapping S on X is a common fixed point of S and the identity mapping I_X on X. Motivated by this fact, Jungck [5] obtained the following extension of Banach Contraction Principles by replacing I_X by a continuos mapping T of X into itself.

THEOREM A. *A contnuous self-mapping T of a complete metric space (X, ρ) has fixed point if and only if there exist $q \in (0,1)$ and a map $S: X \to X$ which commutes with T and satisfies:*

(a). $$S(X) \subset T(X),$$
(b). $$\rho(Sx, Sy) \leq q\rho(Tx, Ty), \text{ for all } x, y \in X.$$

Indeed, S and T have a unique common fixed point.

Further extensions, generalizations and applications of Jungck's. Theorem have been derived by Kasahara [6], Meade and Singh [8], Park ([9], [10], [11]), Park and Park [12], and Ranganathan [13].

It may be remarked that in theorem A, the continuity of the mapping S is a consequence of (b), and the same was used in the proof of Theorem A. Ranganathan [13] observed that Theorem A can be generalized without actually using the continuity of S. The results due to Ranganathan [13] read as follows:

THEOREM B. *Let T be a continuous mapping of a complete metric space (X, ρ) into itself. Then T has a fixed point in X if and only if there exists a real number $q \in (0,1)$ and a mapping $S: X \to X$ which commutes with T and satisfuing*

(a). $$S(X) \subset T(X),$$
(b). $$\rho(Sx, Sy) \leq q \max\{\rho(Tx, Sx), \rho(Ty, Sy), \rho(Tx, Sy), \rho(Ty, Sx), \rho(Tx, Ty)\}$$

for all $x, y \in X$.

Indeed, commuting mappings S and T have a unique common fixed point if (b) holds for some $q \in (0,1)$.

More recently, Ćirić [4] defined a new condition of common contractivity for a pair of mappings of a metrizable space into itself and proved some theorems about common fixed points of family of contractive maps on a uniform space. Following is the main result of Ćirić [4].

THEOREM C. *Let* Y *be a metrizable uniform space and* S *ane* T *be a pair of self-mappings of* Y. *If* (Y,ρ), *for some metric* ρ, *is complete and the mappings* S *and* T *satisfy the condition*

$$\rho(Sx,Ty) \leq q \max\left\{\rho(x,y), \frac{1}{2}, (x,Sx), \frac{1}{2}\rho(y,Ty), \rho(x,Ty), \rho(y,Sx)\right\}$$

for some $q < 1$ *and all* $x, y \in Y$, *then* S *and* T *have a unique common fixed point.* Ćirić [4] used Theorem C to obtain a common fixed point theorem in a sequential complete uniform space.

In this note an attempt has been made to extende Theorem B from metric spaces to uniform spaces which are generalizations of fixed point theorems due to Ćirić [13] and Jungck [5]. Some results on the convergence of sequences of mappings and their fixed point are also presented for mappings satisfying conditions of Theorem B and Theorem C.

2. Preliminaries. Throughout the rest of the paper (X,U) stands for a Hausdorff sequentially complete uniform space. Let P be a fixed family of pseudometrics on X which generates the uniformity U. Following Kelley ([7]), Chapter 6) we define

(i) $$V_{(\rho,r)} = \{(x,y)\colon x,\ y \in X,\ \rho(x,y) < r,\ r > 0\}.$$

(ii) $$G = \left\{V\colon V = \bigcap_{i=1}^{n} V_{(\rho_i,r_i)}, \rho_l \in P, r_i > 0,\ i = 1,2,\dots,n\right\}$$

For $r > 0$,

(iii) $$\alpha V = \left\{\bigcap_{i=1}^{n} V_{(\rho_i \alpha r_i)}\colon \rho_l \in P, r_i > 0, i = 1,2,3,\dots,n\right\},$$

Following results are due to Acharya [1].

LEMMA 2.1. If $V \in G$ *and* $\alpha, \beta > 0$ *then*

(a). $\alpha(\beta V) = (\alpha\beta)V$,

(b). $\alpha V \circ \beta V \subset (\alpha+\beta)V$, (c). *$\alpha V \subset \beta V$ for $\alpha < \beta$.*

LEMMA 2.2. *Let ρ be any pseudo-metric on X, and α, $\beta > 0$. If*

$$(x,y) \in \alpha V_{(\rho,r_1)} \circ \beta V_{(\rho,r_2)}, \text{then } \rho(x,y) < \alpha r_1 + \beta r_2.$$

LEMMA 2.3. *If $x, y \in X$, then for every V in G there is a positive number λ such that $(x,y) \in \lambda V$.*

LEMMA 2.4. *For any arbitrary $V \in G$ there is a pseudo-metric ρ on X such that $V = V_{(\rho,1)}$.*

The pseudo-metric ρ of Lemma 2.4 is called the *Minkowski pseudo-metric* of V.

3. Common Fixed Point Theorems

Before we present our main result we note that the proof of Theorem B can be carried over to obtain the following fihed point theorem in metrizable uniform spaces.

THEOREM 3.1. *Let Y be a metrizable uniform space and S, T be a pair of comuting self-mappings on X such that T is continuous and $S(X) \subset T(X)$. If (Y,ρ), for some metric ρ, is complete and the mappings S and T satisfy*

$$\rho(Sx, Sy \leq) q \max\{\rho(Tx, Sy), \rho(Ty, Sy), \rho(Tx, Sy), \rho(Ty, Sx), \rho(TxTy)\},$$

for all x, y in Y and $q \in (0,1)$, then S and T have a unique common fixed point.

THEOREM 3.2. *Let S and T be two commuting self-mappings of X such that T it continuous, $S(X) \subset T(X)$. If for any $V_i \in G (i = 1,2,3,4,5)$ and x, $y \in X$*

$$(Tx, Sx) \in V_1, \quad (Ty, Sy) \in V_2, \quad (Tx, Sy) \in V_3, \quad (Ty, Sx) \in V_4, \quad (Tx, Ty) \in V_5$$

implies

$$(*) \quad (Sx, Sy) \in a_1V_1 \circ a_2V_2 \circ a_3V_3 \circ a_4V_4 \circ a_5V_5,$$

for some non-negative functions $a_i = a_i(x,y)$, $i = 1,2,3,4,5$ satisfying

$$(a_1 + a_2 + a_3 + a_4 + a_5) \leq q < 1,$$

then S and T have a unique common fixed point.

PROOF. Let x, $y \in X$ and $V \in G$ be arbitrary. Let ρ be the Minkowski pseudo-metric of V. Put $\rho(Tx, Sx) = r_1$, $\rho(Ty, Sy) = r_2$, $\rho(Tx, Sy) = r_3$, $\rho(Ty, Sx) = r_4$, $\rho(Ty, Ty) = r_5$. Let $\varepsilon > 0$. Then

$$(Tx, Sx) \in (r_1 + \varepsilon)V, \quad (Ty, Sy) \in (r_2 + \varepsilon)V, \quad (Tx, Sy) \in (r_3 + \varepsilon)V,$$
$$(Ty, Sx) \in (r_4 + \varepsilon)V, \quad (Tx, Ty) \in (r_5 + \varepsilon)V$$

Therefore by (*) we have

$$(Sx, Sy) \in a_1(r_1+\varepsilon)V \circ a_2(r_2+\varepsilon)V \circ a_3(r_3+\varepsilon)V \circ a_4(r_4+\varepsilon)V \circ a_5(r_5+\varepsilon)V.$$

Hence using Lemma 2.1 (a), Lemma 2.2 and Lemma 2.3 we get

$$\begin{aligned}\rho(Sx, Sy) &< a_1(r_1+\varepsilon) + a_2(r_2+\varepsilon) + a_3(r_3+\varepsilon) + a_4(r_4+\varepsilon) + a_5(r_5+\varepsilon)\\ &= a_1\rho(Tx, Sy) + a_2\rho(Ty, Sy) + a_3\rho(Tx, Sy) + a_4\rho(Ty, Sx)\\ &\quad + a_5\rho(Tx, Ty) + \left(\sum_{i=1}^{5} a_i\right)\varepsilon.\end{aligned}$$

As ε is arbitrary,

$$\begin{aligned}\rho(Sx, Sy) &\le a_1\rho(Tx, Sx) + a_2\rho(Ty, Sy) + a_3\rho(Tx, Sy) + a_4\rho(Ty, Sx) + a_5\rho(Tx, Ty)\\ &\le \left(\sum_{i=1}^{5} a_i\right)\max\{\rho(Tx, Sx), \rho(Ty, Sy), \rho(Tx, Sy), \rho(Ty, Sx), \rho(Tx, Ty)\}\\ &\le q\cdot\max\{\rho(Tx, Sx), \rho(Ty, Sy), \rho(Tx, Sy), \rho(Ty, Sx), \rho(Tx, Ty)\}.\end{aligned}$$

Then by an argument similar to the one used in the proof of theorem B, we obtain that $\rho(u, Su) = \rho(u, Tu) = 0$ for some $u \in X$.

Therefore $(u, Su) \in V$ and $(u, Tu) \in V$ for every $V \in G$. This shows that $u = Su = Tu$. Uniqueness of the common fixed point u of S and T is not difficult to prove (*cf.* Acharya [1]). This completes the proof.

Remark. Our Theorem 3.2 is an extended version of theorem A in uniform spaces.

Corollary 3.3. *Let S be a self-mapping on X such for $V_i \in G$ $(i = 1, 2, 3, 4, 5)$ and $x, y \in X$*

$$(x, Sx) \in V_1, \quad (y, Sy) \in V_2, \quad (x, Sy) \in V_3, \quad (y, Sx) \in V_4 \text{ and } (x, y) \in V_5$$

implies

$$(Sx, Sy) \in a_1V_1 \circ a_2V_2 \circ a_3V_3 \circ a_4V_4 \circ a_5V_5$$

where a_i are non-negative numbers with $\sum_{i=1}^{5} a_i \le q < 1$. Then S has a unique fixed point.

Remark. Corollary 3.3 may be regarded as the extension of Ćirić's fixed point theorem [3] from metric spaces to uniform spaces. This Corollary also generalizes Theorem 3.1 of Acharya [1].

Corollary 3.4. *Let T be a continuous mapping of X into itself. Let F be a family of self-mappings on X each of which commutes with T and $T^*(X) \subset T(X)$ for each $T^* \in F$. If there exists some $S \in F$ such that for each $T^* \in F$ there is* *a positive integer* $k = k(T^*)$ *such that* S^k *and* T^* *satisfy condition* $(*)$ *of Theorem 3.2 then* F *has a unique common fixed point.*

COROLLARY 3.5. *Let* S *be a mapping of* X *onto itself such that for a positive integer* n, S^{n+1} *is continuous. If for any* $V_i \in G$ $(i = 1, 2, 3, 4, 5)$ *and* x, y *in* X

$$(S^n x, x) \in V_1, \quad (S^n y, y) \in V_2, \quad (S^n x, y) \in V_3, \quad (S^n y, x) \in V_4, \quad (S^n x, S^n y) \in V_5$$

implies that

$$(x, y) \in a_1 V_1 \circ a_2 V_2 \circ a_3 V_3 \circ a_4 V_4 \circ a_5 V_5$$

for some non-negative functions $a_i = a_i(x, y)$, *with* $a_i < \frac{1}{5}$ *for each* $i = 1, 2, 3, 4, 5$, *then* S *and* T *have a unique common fixed point.*

4. Convergence Theorems. Let us call the pair (S, T) of mappings S and T satisfying all the hypotheses of Theorem 3.2 (Theorem 3.1) as a Jungck's quasi-contraction pair on the uniform space X (metric space (Y, ρ)). If (S, T) satisfies all hypotheses of Theorem C, we shall call (S, T) a Ćirić's contractive pair on the matric space (Y, ρ). Note that when $T = I_X$, S becomes quasi-contraction in the sense of Ćirić [3]. Now we wish to prove convergence theorems concerning the sequences of mappings and their fixed points in uniform spaces (cf. Acharya [21]).

THEOREM 4.1. *Let* Y *be a metrizable uniform space such that for some metric* ρ, (Y, ρ) *is complete. Let* $\{S_n\}$ *and* $\{T_n\}$ *be two sequences of self-mappings on* Y *such that* (S_n, T_n) *is a Jungck's quasi-contraction pair on* (Y, ρ) *for each* n. *If* S *are* T *are the pointwise limit of* $\{S_n\}$ *and* $\{T_n\}$, *respectively, then* (S, T) *is a Jungck's quasi-contraction pair on* (Y, ρ). *Furthermore if* $q < \frac{1}{2}$ *the sequence of unique common fixed points of* S_n *and* T_n *converges to the unique common fixed point of* S *and* T.

PROOF. Let x, y be arbitrary elements of X. Then we have

$$\begin{aligned}
\rho(Sx, Sy) \leq & \rho(Sx, S_n x) + \rho(S_n x, S_n y) + \rho(S_n y, Sy) \\
\leq & \rho(Sx, S_n x) + \rho(S_n y, Sy) + q \max\{\rho(T_n x, S_n x), \rho(T_n y, S_n y), \\
& \rho(T_n x, S_n y), \rho(T_n y, S_n x), \rho(T_n x, T_n y)\} \\
\leq & \rho(Sx, S_n x) + \rho(S_n y, Sy) + q \max\{\rho(T_n x, Tx) \rho(Tx, Sx) + \rho(Sx, S_n x), \\
& \rho(T_n y, Ty) + \rho(Ty, Sy) + \rho(Sy, S_n y), \\
& \rho(T_n x, Tx) + \rho(Tx, Sy) + \rho(Sy, S_n y), \\
& \rho(T_n y, Ty) + \rho(Ty, Sx) + \rho(Sx, S_n y), \\
& \rho(T_n x, Tx) + \rho(Tx, Ty) + \rho(Ty, T_n y)\}
\end{aligned}$$

As $S_n x \to Sx$, $S_n y \to S_y$, $T_n x \to T_x$, $T_n y \to T_y$ when $n \to \infty$, we have

$$\rho(Sx, Sy) \leq q \max\{\rho(Tx, Sx), \rho(Ty, Sy), \rho(Tx, Sy), \rho(Ty, Sx), \rho(Tx, Ty)\}.$$

Also $S_n(Y) \subset T_n(Y)$ for each n implies that $S(Y) \subset T(Y)$. Therefore (S,T) is a Jungck's quasi-contraction pair on (Y,ρ) and there exists a unique common fixed point u of S and T since (Y,ρ) is complete.

Let $\{u_n\}$ be the unique common fixed point of S_n and T_n for each n. Since S and T are pointwise limits of $\{S_n\}$ and $\{T_n\}$, respectively, for every $\varepsilon > 0$, there are positive integers N_1 and N_2 such that

$$\rho(S_n u, u) = \rho(S_n u, Su) < \min\left\{\frac{1-q}{1-2q}, \frac{1}{1-q}\right\}\frac{\varepsilon}{2}, \quad \text{for } n \geq N_1,$$

and

$$\rho(T_n u, u) = \rho(T_n u, Tu) < \min\left\{\frac{1-2q}{q}, \frac{1-q}{q}\right\}\frac{\varepsilon}{2}, \quad \text{for } n \geq N_2.$$

First we which to estimate the distance $\rho(S_n u_n, S_n u)$. For this we note that

$$\begin{aligned}\rho(S_n u_n, S_n u) \leq q \max\{&\rho(T_n u_n, S_n u_n), \rho(T_n u, S_n u), \rho(T_n u_n, S_n u),\\ &\rho(T_n u, S_n u_n), \rho(T_n u_n, T_n u)\}\\ \leq q \max\{&\rho(T_n u, S_n u), \rho(T_n u, S_n u_n), \rho(T_n u_n, T_n u)\}.\end{aligned}$$

Considering all three cases, we have

$$\rho(S_n u_n, S_n u) \leq \frac{q}{1-q}\{q(T_n u, u) + \rho(u_n, u)\}$$

or

$$\rho(S_n u_n, S_n u) \leq q\{\rho(T_n u, u) + \rho(u_n, u)\}.$$

Then by

$$\rho(u_n, u) \leq \rho(S_n u_n, S_n u) + \rho(S_n u, u),$$

we have

$$\rho(u_n, u) \leq \left(\frac{q}{1-2q}\right)\rho(T_n u, u) + \left(\frac{1-q}{1-2q}\right)\rho(S_n u, u),$$

or

$$\rho(u_n, u) \leq \left(\frac{q}{1-q}\right)\rho(T_n u, u) + \left(\frac{1}{1-q}\right)\rho(S_n u, u).$$

In both of the cases we get

$$\rho(u_n, u) < \varepsilon \quad \text{for} \quad u \geq \max(N_1, N_2).$$

Hence $\{u_n\}$ converges to u.

Remark. The constant q for the pair (S_n, T_n) in Theorem 4.1 can be replaced by a sequence of constants q_n such that $g_n \to q < \frac{1}{2}$ where q is the constant for the pair (S,T).

Now we state the uniform space version of Theorem 4.1 which can be proved by the method used in the proof of Theorem 3.2.

THEOREM 4.2. *Let $\{S_n\}$ and $\{T_n\}$ be two sequences of self-mappings on X such that for each n, limit (S_n, T_n) is a Jungck's quasi-contraction pair on X. If S and T are the pointwise limit of $\{S_n\}$ and $\{T_n\}$ respectively, such that (S,T) is a Jungck's quasi-contraction pair on X, then the sequence $\{u_n\}$ of unique common fixed points of S_n and T_n converges to the unique common fixed point u of S and T.*

We can also prove the following:

THEOREM 4.3. *Let $\{S_n\}$ and $\{T_n\}$ be two sequences of self-mappings on a metrizable uniform space Y which is complete with respect to some metric ρ. If S_n and T_n converges uniformly to self-mappings S and T on Y, respectively, such that (S,T) is a Jungck's quasi-contraction pair on (Y, ρ) the sequence $\{u_n\}$ of common fixed points of S_n and T_n (provided u_n exists for each n) converges to the unique common fixed point u of S and T.*

PROOF. Firstly, we hawe

$$\begin{aligned}\rho(Su_n, Su) \leq & q\max\{\rho(Tu_n, Su_n), \rho(Tu, Su), \rho(Tu_n, Su), \rho(Tu, Su_n), \rho(Tu_n, Tu)\}\\ = & q\max\{\rho(Tu_n, Su_n), \rho(Tu_n, u), \rho(Su, Su_n), \rho(Tu_n, u)\}\\ = & q\max\{\rho(Tu_n, Su_n), \rho(Tu_n, u)\}\end{aligned}$$

Then using

$$\rho(u_n, u) \leq \rho(S_n u_n, Su_n) + \rho(Su_n, Su),$$

we have

$$\rho(u_n, u) \leq q\rho(T_n u_n, Tu_n) + (1+q)\rho(S_n u_n, Su_n),$$

or

$$\rho(u_n, u) \leq \left(\frac{q}{1-q}\right)\rho(T_n u_n T u_n).$$

In both the cases , we find that $u_n \to u$, completing the proof.

Following is the uniform space version of Theorem 4.3.

THEOREM 4.4. *Let $\{S_n\}$ and $\{T_n\}$ be two sequences of self-mappings on X. If S and T are the uniform limits of $\{S_n\}$ and $\{T_n\}$, respectively, such that (S,T) is a Jungck's quasi-contraction pair on X, the sequence $\{u_n\}$ of common fixed points of S_n and T_n (provided u_n exists for each n) converges to the unique common fixed point of S and T.*

The next result can be proved by the method of Theorem 4.1.

THEOREM 4.5. *Let Y be a metrizable uniform space such that for some metric ρ, (Y, ρ) is complete. Let $\{S_n\}$ and $\{T_n\}$ be two sequnces of self-mappings on Y such that (S_n, T_n) is a Ćirić's contractive pair on (Y, ρ) for each n. If S_n and*

T_n converge pointwise to self-mappings S and T on Y, $\rho)$, respectively, then (S,T) is a Ćirić conractive pair on (Y,ρ). Furthermore, the sequence $\{u_n\}$ of unique common fixed point of S_n and T_n converges to the unique common fixed point of S and T.

THEOREM 4.6. *Let $\{S_n\}$ and $\{T_n\}$ be two sequences of self-mappings on a metrizable uniform space Y which is complete with respect to some metric ρ. Suppose that $\{u_n\}$ and $\{v_n\}$ are the sequences of fixed points of $\{S_n\}$ and $\{T_n\}$, respectively. If S and T are the uniform limits of $\{S_n\}$ and $\{T_n\}$ such that (S,T) is a Ćirić contractive pair on (Y,ρ) and x_0 is the unique common fixed point of S and T, then both the sequences $\{u_n\}$ and $\{v_n\}$ converge to x_0.*

PROOF. $\rho(u_n, x_0) = \rho(S_n u_n, Tx_0) \leq \rho(S_n u_n, Su_n) + \rho(Su_n, Tx_0)$.
But

$$\begin{aligned}\rho(Su_n, Tx_0) &\leq q \max\left\{\rho(u_n, x_0), \frac{1}{2}\rho(x_0, Tx_0), \frac{1}{2}\rho(u_n, S_n u_n), \rho(u_n Tx_0), \rho(x_0, S_n u_n)\right\}\\ &= q\rho(u_n, x_0).\end{aligned}$$

Thus we have

$$\rho(u_n, x_0) \leq \left(\frac{1}{1-q}\right)\rho(S_n u_n, Su_n),$$

which shows that $u_n \to x_0$.

Similarly, we can prove that $\{v_n\}$ also converges to x_0.

REMARK. If $u_n = v_n$ for each n, then Theorem 4.6. says that the sequnce of common fixed points of S_n and T_n converges to the unique common fixed point of S and T. (cf. our Theorem 4.3).

THEOREM 4.7. *Let Y be a metrizable uniform space which is complete for some metric ρ. Let $\{S_n\}$ and $\{T_n\}$ be sequences of self-mappings on Y such that S_n and T_n have a common fixed point u_n for each n. Let $\{S_n\}$ and $\{T_n\}$ converge uniformly to self-mappings S and T on Y such that (S,T) is a Ćirić's contractive pair on (Y,ρ). If $\{u_n\}$ contains a subsequence $\{U_{n_i}\}$ convering to u_0, then u_0 is a unique common fixed point of S and T.*

PROOF. Since (S,T) is a Ćirić's contractive pair we have:

$$\begin{aligned}\rho(Su_{n_i}, Tu_0) \leq q \max\Big\{&\rho(u_{n_i}, u_0), \frac{1}{2}\rho(u_{n_i}, Su_{n_i}), \frac{1}{2}\rho(u_0, Tu_0),\\ &\rho(u_{n_i}, Tu_0), \rho(u_0, Su_{n_i})\Big\}.\end{aligned}$$

Form this one gets *one* of the following:

(i) $$\rho(Su_{n_i}, Tu_0) \leq q\rho(U_{n_i}, u_0),$$

(ii) $$\rho(Su_{n_i}, Tu_0) \leq \left(\frac{2q}{2-q}\right)\rho(u_{n_i}, Tu_0),$$

(iii) $$\rho(Su_{n_i}, Tu_0) \leq \frac{q}{2}[\rho(u_0, u_{n_i}) + \rho(u_{n_i}, Tu_0)],$$

(iv) $$\rho(Su_{n_i}, Tu_0) \leq \rho(u_{n_i}, Tu_0),$$

(v) $$\rho(Su_{n_i}, Tu_0) \leq q[\rho(u_0, u_{n_i}) + \rho(S_{n_i}u_{n_i}, Su_{n_i}].$$

Using any of the above relations and the inequality

$$\rho(u_{n_i}, Tu_0) \leq \rho(S_{n_i}u_{n_i}, Su_{n_i}) + \rho(Su_{n_i}, Tu_0),$$

we see that $u_{n_i} \to Tu_0$. Therefore $u_0 = Tu_0$. Similarly, we can show that u_0 is also a fixed point of S. Clearly, u_0 is unique since (S, T) is a Ćirić's contractive pair. This completes the proof.

We remark that Theorems 4.5–4.7 also a hold good when stated for sequential complete uniform spaces. (cf. Theorem 2, Ćirić [4]).

Finally, we also note that should we solve the problem posed by Ćirić [4] at the end of his paper, one can prove convergence theorems for this new result as well.

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