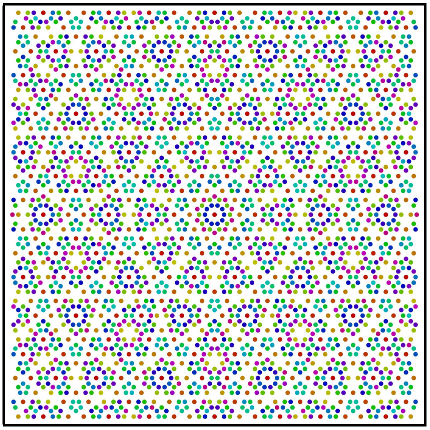
Quasilattice principle:
For a quasicrystal point set in n-dimensional space, the points that lie on any straight line can be mapped by an affine transformation to a valid quasicrystal sequence in 1-dimensional space.



Star map:
$(xe_1 + ye_2)^* = x'e_1^* + y'e_2^*$ semilinear map acts on golden integers $x, y \in \mathbb{Z}[\tau]$ and basis vectors $e_1, e_2 \in \mathbb{R}^2$.

Star basis:
$e_1 = (0,1)$ and $e_2 = (-\frac{1}{2}\tau, \frac{1}{2}\sqrt{3-\tau})$ are mapped by the star map to $e_1^* = (0,1)$ and $e_2^* = (\frac{1}{2}(\tau-1), -\frac{1}{2}\sqrt{2+\tau})$, as defined by the standard basis of the non-crystallographic Coxeter group H_2.

Golden integer lattice:
$M = \mathbb{Z}[\tau]e_1 + \mathbb{Z}[\tau]e_2 = \left\{ (a_1 + b_1\tau)e_1 + (a_2 + b_2\tau)e_2 \,\middle|\, a_1, b_1, a_2, b_2 \in \mathbb{Z} \right\}$ is a $\mathbb{Z}[\tau]$-module that is dense in $\mathbb{R}^2$.

Star mapped golden integer lattice:
$M^* = \mathbb{Z}[\tau']e_1^* + \mathbb{Z}[\tau']e_1^* = \left\{ a_1 + b_1\tau')e_1^* + (a_2 + b_2\tau')e_1^* \,\middle|\, a_1, b_1, a_2, b_2 \in \mathbb{Z} \right\}$ is determined by the star map.

Cut-and-project 2D quasicrystals:
$\Sigma_\Omega = \left\{ xe_1 + ye_2 \in M \,\middle|\, x'e_1^* + y'e_2^* \in \Omega \cap M^* \right\}$ quasicrystal is specified by a bounded acceptance window Ω.

Duality of 2D quasicrystals:
$xe_1 + ye_2 \in (\Sigma_\Omega \cap V) \subset M$ restricted to the bounded viewing window region V implies a dual quasicrystal $x'e_1^* + y'e_2^* \in (\Sigma_V^* \cap \Omega) \subset M^*$ contained in the bounded acceptance window region Ω.

Set laws of 2D quasicrystals:
$\Sigma_{\Omega_1 \cap \Omega_2} = \Sigma_{\Omega_1} \cap \Sigma_{\Omega_2}$ and $\Sigma_{\Omega_1 \cup \Omega_2} = \Sigma_{\Omega_1} \cup \Sigma_{\Omega_2}$ as well as $\Sigma_{\Omega_1} \subseteq \Sigma_{\Omega_2}$ whenever $\Omega_1 \subseteq \Omega_2$.

Translation and scaling of 2D quasicrystals:
$\Sigma_\Omega + \lambda = \Sigma_{\Omega+\lambda^*}$ for $\lambda \in M$ and $\xi'\Sigma_\Omega = \Sigma_{\xi\Omega}$ for $\xi = \tau^k$ and any $k \in \mathrm{Z}$.

Inflation symmetry of 2D quasicrystals:
$\tau^2(\Sigma_\Omega - z) + z \subset \Sigma_\Omega$ implies that every point $z \in \Sigma_\Omega$ is a center of inflation symmetry, if Ω is a convex set.

Quasilattice for 2D quasicrystals:
$\Sigma_{\Gamma_1 e_1^* + \Gamma_2 e_2^*} = \Sigma_{\Gamma_1} e_1 + \Sigma_{\Gamma_2} e_2$ is a lattice of 1D quasicrystals Σ_{Γ_1} and Σ_{Γ_2} with acceptance windows Γ_1 and Γ_2.

Quasilattice algorithm for 2D quasicrystals:

1. Find a quasilattice viewing window $W = W_1 e_1 + W_2 e_2$ that contains the desired quasicrystal viewing window $V \subseteq W$. The quasilattice viewing window W is a parallelogram where the viewing intervals satisfy $x \in W_1$ and $y \in W_2$ for all $xe_1 + ye_2 \in V \cap M$.
2. Find a quasilattice acceptance window $\Gamma = \Gamma_1 e_1^* + \Gamma_2 e_2^*$ that contains the desired quasicrystal acceptance window $\Omega \subseteq \Gamma$. The quasilattice acceptance window Γ is a parallelogram where the acceptance intervals satisfy $x' \in \Gamma_1$ and $y' \in \Gamma_2$ for all $x'e_1^* + y'e_2^* \in \Omega \cap M^*$.
3. Generate the 1D quasicrystals $\Sigma_{\Gamma_1} \cap W_1$ and $\Sigma_{\Gamma_2} \cap W_2$ according to viewing intervals W_1 and W_2 as well as the acceptance intervals Γ_1 and Γ_2.
4. Generate the 2D quasilattice $\Sigma_\Gamma \cap W = \Sigma_{\Gamma_1 e_1^* + \Gamma_2 e_2^*} \cap W = (\Sigma_{\Gamma_1} \cap W_1)\, e_1 + (\Sigma_{\Gamma_2} \cap W_2)\, e_2$ according to quasilattice viewing window W and quasilattice acceptance window Γ from the coordinates supplied by the 1D quasicrystals $\Sigma_{\Gamma_1} \cap W_1$ and $\Sigma_{\Gamma_2} \cap W_2$.
5. Discard all quasilattice points $xe_1 + ye_2 \in \Sigma_\Gamma \cap W$ that do not belong to the desired quasicrystal $xe_1 + ye_2 \notin \Sigma_\Omega \cap V$ because either they do not belong to the desired quasicrystal viewing window $xe_1 + ye_2 \notin V$ or acceptance window $x'e_1^* + y'e_2^* \notin \Omega$.