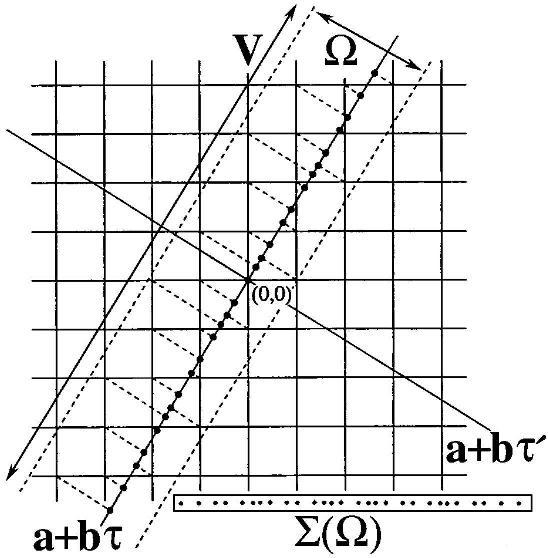
Cut-and-project principle:
To generate a non-periodic point set in an n-dimensional space, take a region of a periodic point set in a $2n$-dimensional space and orthogonally project it onto an irrationally oriented n-dimensional subspace. For the resulting point set in n-dimensional space to be discrete rather than dense, the region of $2n$-dimensional space undergoing projection, typically a cylinder, must be bounded along the directions of projection.



Ruler and graph paper construction:
Start with the 2D integer lattice. Consider three parallel lines with slope τ, the inner one passing through the origin $(0,0)$. The outer lines serve to cut out a strip of the integer lattice, while the inner line provides the central axis. The width of the strip defines the acceptance window $\Omega = [c, d)$, while the visible length of the strip defines the viewing window $V = [l, r]$. Observe that when the lattice points contained in the strip are orthogonally projected onto the central axis, the resulting 1D sequence Σ_Ω is non-periodic if and only if the slope τ of the strip is irrational.

Golden ratio:
$\tau = \frac{1}{2}\left(1+\sqrt{5}\right) \approx 1.618$ and its conjugate $\tau' = \frac{1}{2}\left(1-\sqrt{5}\right) \approx -0.618$ are the solutions of $x^2 = x + 1$.

Golden integers:
$\mathbb{Z}[\tau] = \{a + b\tau \mid a, b \in \mathbb{Z}\}$ is an Euclidean domain that is dense in $\mathbb{R}$.

Conjugate golden integers:
$\mathbb{Z}[\tau'] = \{a + b\tau' \mid a, b \in \mathbb{Z}\}$ is the set of conjugates $(a+b\tau)' = a + b\tau' = a - b\tau^{-1}$, where $\tau' + \tau = 1$ and $\tau'\tau = -1$.

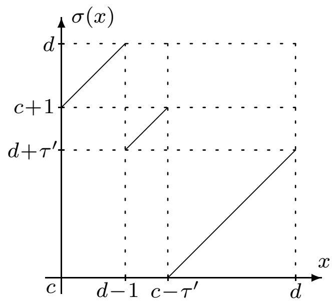
Cut-and-project 1D quasicrystals:
$\Sigma_\Omega = \left\{ a + b\tau \in \mathbb{Z}[\tau] \,\middle|\, a + b\tau' \in \Omega \cap \mathbb{Z}[\tau'] \right\}$ quasicrystal is specified by a bounded acceptance window $\Omega = [c, d)$.

Duality of 1D quasicrystals:
$x_k \in (\Sigma_\Omega \cap V) \subset \mathbb{Z}[\tau]$ restricted to the bounded viewing interval $V = [l, r]$ implies a dual quasicrystal
$x'_k \in (\Sigma'_V \cap \Omega) \subset \mathbb{Z}[\tau']$ contained in the bounded acceptance interval $\Omega = [c, d)$.

Translation and scaling of 1D quasicrystals:
$\Sigma_{[c,d)} + \lambda = \Sigma_{[c+\lambda', d+\lambda')}$ for $\lambda \in \mathbb{Z}[\tau]$ and $\xi'\Sigma_{[c,d)} = \Sigma_{[\xi c, \xi d)}$ for $\xi = \tau^k$ and $k \in \mathbb{Z}$.

Stepping function for 1D quasicrystals:
Assume a standard acceptance window $\Omega = [c, d)$ such that $0 \in \Omega$ and $d - c \in [1, \tau)$ and observe that a 1D quasicrystal is an arrangement of just three possible tiles.

$$x'_{k+1} = \begin{cases} x'_k + 1 & \text{if } x'_k \in [c, d-1) \quad \Rightarrow \quad x_{k+1} - x_k = 1 \text{ (small tile)} \\ x'_k + 1 + \tau' & \text{if } x'_k \in [d-1, c-\tau') \quad \Rightarrow \quad x_{k+1} - x_k = 1 + \tau \text{ (large tile)} \\ x'_k + \tau' & \text{if } x'_k \in [c-\tau', d) \quad \Rightarrow \quad x_{k+1} - x_k = \tau \text{ (medium tile)} \end{cases}$$



Iterative construction of 1D quasicrystals:
For a standard acceptance window $\Omega = [c, d)$, containing the origin $c \le 0 < d$ inside an interval of suitable width $1 \le d - c < \tau$, a quasicrystal sequence $x_k \in \Sigma_\Omega$ is generated by successively applying the stepping function $\sigma : \Omega \to \Omega$ to obtain the conjugate of the right neighbor $x'_{k+1} = \sigma(x'_k)$ or the inverse stepping function $\sigma^{-1} : \Omega \to \Omega$ to obtain the conjugate of the left neighbor $x'_{k-1} = \sigma^{-1}(x'_k)$.

Stepping algorithm for 1D quasicrystals:

1. Translate and scale the desired acceptance Ω and viewing V intervals to make the acceptance window Ω into a standard acceptance window.
2. Starting from $x_0 = 0$, apply the stepping function $x'_{k+1} = \sigma(x'_k)$ and its inverse $x'_{k-1} = \sigma^{-1}(x'_k)$ to generate the consecutive quasicrystal points until both edges of the translated and scaled viewing window are reached.
3. Reverse the translation and scaling to obtain the desired quasicrystal Σ_Ω.