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UPPER AND LOWER BOUNDS OF SOLUTIONS FOR FRACTIONAL INTEGRAL EQUATIONS

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Abstract. In this paper we consider the integral equation of fractional order in sense of Riemann-Liouville operator

$$u^m(t) = a(t)I^{\alpha}[b(t)u(t)] + f(t)$$

with $m \geq 1$, $t \in [0,T]$, $T < \infty$ and $0 < \alpha < 1$. We discuss the existence, uniqueness, maximal, minimal and the upper and lower bounds of the solutions. Also we illustrate our results with examples.

Full text

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