ON THE PERIODIC MILD SOLUTIONS TO COMPLETE HIGHER ORDER DIFFERENTIAL EQUATIONS ON BANACH SPACES

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Abstract. For the complete higher order differential equation

$$u^{(n)}(t) = \sum_{k=0}^{n-1} A_k u^{(k)}(t) + f(t), \qquad 0 \le t \le T,$$

on a Banach space E, we give necessary and sufficient conditions for the periodicity of mild solutions. The results, which are proved in a simple manner, generalize some well-known ones.

Full text

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