

**BOUNDARY VALUE PROBLEMS FOR
FRACTIONAL DIFFERENTIAL INCLUSIONS WITH
FOUR-POINT INTEGRAL BOUNDARY
CONDITIONS**

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Abstract. In this paper, we discuss the existence of solutions for a boundary value problem of second order fractional differential inclusions with four-point integral boundary conditions involving convex and non-convex multivalued maps. Our results are based on the nonlinear alternative of Leray Schauder type and some suitable theorems of fixed point theory.

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References

- [1] B. Ahmad, *Existence of solutions for irregular boundary value problems of nonlinear fractional differential equations*, Appl. Math. Lett. **23** (2010), 390-394. [MR2594849](#) (2010m:34004). [Zbl 1198.34007](#).
- [2] B. Ahmad and J. R. Graef, *Coupled systems of nonlinear fractional differential equations with nonlocal boundary conditions*, Panamer. Math J. **19** (2009), 29-39. [Zbl 1180.34002](#).
- [3] B. Ahmad and J. J. Nieto, *Existence results for a coupled system of nonlinear fractional differential equations with three-point boundary conditions*, Comput. Math. Appl. **58** (2009), 1838-1843. [MR2557562](#) (2010j:34033). [Zbl 1205.34003](#).
- [4] B. Ahmad and V. Otero-Espinar, *Existence of solutions for fractional differential inclusions with anti-periodic boundary conditions*, Bound. Value Probl. 2009, Art. ID 625347, 11 pp. [MR2525570](#) (2010e:34014). [Zbl 1172.34004](#).

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<http://www.utgjiu.ro/math/sma>

- [5] A. Bressan and G. Colombo, *Extensions and selections of maps with decomposable values*, Studia Math. **90** (1988), 69-86. [MR0947921](#) (89j:54021). [Zbl 0677.54013](#).
- [6] C. Castaing and M. Valadier, *Convex Analysis and Measurable Multifunctions*, Lecture Notes in Mathematics **580**, Springer-Verlag, Berlin-Heidelberg-New York, 1977. [MR0467310](#) (57#7169). [Zbl 0346.46038](#).
- [7] H. Covitz and S. B. Nadler Jr., *Multivalued contraction mappings in generalized metric spaces*, Israel J. Math. **8** (1970), 5-11. [MR0263062](#) (41#7667). [Zbl 0192.59802](#).
- [8] V. Daftardar-Gejji and S. Bhalekar, *Boundary value problems for multi-term fractional differential equations*, J. Math. Anal. Appl. **345** (2008), 754-765. [MR2429175](#) (2009i:35173). [Zbl 1151.26004](#).
- [9] M.A. Darwish and S.K. Ntouyas, *On initial and boundary value problems for fractional order mixed type functional differential inclusions*, Comput. Math. Appl. **59** (2010), 1253-1265. [MR2579487](#) (2010h:34030). [Zbl 1189.34029](#).
- [10] K. Diethelm and A. D. Freed, *On the solution of nonlinear fractional order differential equations used in the modeling of viscoplasticity*, in "Scientific Computing in Chemical Engineering II-Computational Fluid Dynamics, Reaction Engineering and Molecular Properties" (F. Keil, W. Mackens, H. Voss, and J. Werther, Eds), pp 217-224, Springer-Verlag, Heidelberg, 1999.
- [11] V. Gafiyuchuk, B. Datsko and V. Meleshko, *Mathematical modeling of different types of instabilities in time fractional reaction-diffusion systems*, Comput. Math. Appl. **59** (2010), 1101-1107. [MR2579475](#) (2010i:35426). [Zbl 1189.35151](#).
- [12] L. Gaul, P. Klein and S. Kempfle, *Damping description involving fractional operators*, Mech. Systems & Signal Processing **5** (1991), 81-88.
- [13] S. Hamani, M. Benchohra and J.R. Graef, *Existence results for boundary-value problems with nonlinear fractional differential inclusions and integral conditions*, Electron. J. Differential Equations 2010, No. **20**, 16 pp. [Zbl 1185.26010](#).
- [14] Sh. Hu and N. Papageorgiou, *Handbook of Multivalued Analysis, Theory I*, Kluwer, Dordrecht, 1997. [MR1485775](#) (98k:47001). [Zbl 0887.47001](#).
- [15] A. Granas and J. Dugundji, *Fixed Point Theory*, Springer-Verlag, New York, 2005. [MR1987179](#) (2004d:58012). [Zbl 1025.47002](#).
- [16] M. Kisielewicz, *Differential Inclusions and Optimal Control*, Kluwer, Dordrecht, The Netherlands, 1991. [MR1135796](#) (93c:49001). [Zbl 0731.49001](#).

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<http://www.utgjiu.ro/math/sma>

- [17] A. A. Kilbas, H. M. Srivastava and J. J. Trujillo, *Theory and Applications of Fractional Differential Equations*, North-Holland Mathematics Studies, 204. Elsevier Science B.V., Amsterdam, 2006. [MR2218073](#) (2007a:34002). [Zbl 1092.45003](#).
- [18] V. Lakshmikantham, S. Leela and J. Vasundhara Devi, *Theory of Fractional Dynamic Systems*, Cambridge Academic Publishers, Cambridge, 2009. [Zbl 1188.37002](#).
- [19] A. Lasota and Z. Opial, *An application of the Kakutani-Ky Fan theorem in the theory of ordinary differential equations*, Bull. Acad. Polon. Sci. Ser. Sci. Math. Astronom. Phys. **13** (1965), 781-786. [MR0196178](#) (33#4370). [Zbl 0151.10703](#).
- [20] F. Mainardi, *Fractional calculus: Some basic problems in continuum and statistical mechanics*, in "Fractals and Fractional Calculus in Continuum Mechanics" (A. Carpinteri and F. Mainardi, Eds), pp. 291-348, Springer-Verlag, Wien, 1997. [MR1611587](#) (99f:26010)
- [21] F. Metzler, W. Schick, H. G. Kilian and T. F. Nonnenmacher, *Relaxation in filled polymers: A fractional calculus approach*, J. Chem. Phys. **103** (1995), 7180-7186.
- [22] A. Ouahab, *Some results for fractional boundary value problem of differential inclusions*, Nonlinear Anal. **69** (2008), 3877-3896. [MR2463341](#) (2009h:34014). [Zbl 1169.34006](#).
- [23] I. Podlubny, *Fractional Differential Equations*, Academic Press, San Diego, 1999. [MR1658022](#) (99m:26009). [Zbl 0924.34008](#).
- [24] J. Sabatier, O.P. Agrawal and J. A. T. Machado (Eds.), *Advances in Fractional Calculus: Theoretical Developments and Applications in Physics and Engineering*, Springer, Dordrecht, 2007. [Zbl 1116.00014](#).
- [25] S. G. Samko, A. A. Kilbas and O. I. Marichev, *Fractional Integrals and Derivatives, Theory and Applications*, Gordon and Breach, Yverdon, 1993. [MR1347689](#)(96d:26012). [Zbl 0818.26003](#).

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