

DIFFERENT VERSIONS OF THE IMPRIMITIVITY THEOREM

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Abstract. In this paper we present different versions of the imprimitivity theorem hoping that this might become a support for the ones who are interested in the subject. We start with Mackey's theorem [26] and its projective version [29]. Then we remind Mackey's fundamental imprimitivity theorem in the bundle context [14]. Section 5 is dedicated to the imprimitivity theorem for systems of G -covariance [6]. In Section 6 and 7 we refer to the imprimitivity theorem in the context of C^* -algebras [39] and to the symmetric imprimitivity theorem [36], [42], [11].

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