

EXPLICIT STABILITY CONDITIONS FOR NEUTRAL TYPE VECTOR FUNCTIONAL DIFFERENTIAL EQUATIONS. A SURVEY

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Abstract. This paper is a survey of the recent results of the author on the stability of linear and nonlinear neutral type functional differential equations. Mainly, vector equations are considered. In particular, equations whose nonlinearities are causal mappings are investigated. These equations include neutral type, ordinary differential, differential-delay, integro-differential and other traditional equations. Explicit conditions for the Lyapunov, exponential, input-to-state and absolute stabilities are derived. Moreover, solution estimates for the considered equations are established. They provide us the bounds for the regions of attraction of steady states. A part of the paper is devoted to the Aizerman type problem from the the absolute stability theory. The main methodology presented in the paper is based on a combined usage of the recent norm estimates for matrix-valued functions with the generalized Bohl - Perron principle, positivity conditions for fundamental solutions of scalar equations and properties of the so called generalized norm

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