Surveys in Mathematics and its Applications

ISSN 1842-6298 (electronic), 1843-7265 (print)
Volume 11 (2016), 93 – 106

HEREDITY IN FUNDAMENTAL LEFT COMPLEMENTED ALGEBRAS

Marina Haralampidou and Konstantinos Tzironis

Abstract. In the present paper, we introduce the notion of a fundamental complemented linear space, through continuous projections. This notion is hereditary. Relative to this, we prove that if a certain topological algebra is fundamental, then a concrete subspace is fundamental too. For a fundamental complemented linear space, we define the notion of continuity of the complementor. In some cases, we employ a generalized notion of complementation, that of (left) precomplementation. In our main result, the continuity of the complementor for a certain fundamental complemented (topological) algebra is inherited to the induced vector complementor of the underlying linear space of a certain right ideal. Weakly fundamental algebras are also considered in the context of locally convex ones.

Full text

Acknowledgement. The authors thank the referee for a careful reading of the paper and for several remarks.

References

[1] F. E. Alexander, *Representation theorems for complemented algebras*, Trans. Amer. Math. Soc., **148**(1970), 385–398. MR0275159 (43 #916). Zbl 0196.14901.

[2] Ph. Blanchard, E. Brüning, *Mathematical Methods in Physics. Distributions, Hilbert Space Operators, and Variational Methods*, Birkhäuser, 2003. MR1936762 (2004c:46001). Zbl Zbl 0941.46001.

[3] J. R. Giles, *Introduction to the Analysis of Normed Linear Spaces*, Cambridge University Press, 2000. MR1745287 (2000m:46001). Zbl Zbl 0941.46001.

[4] M. Haralampidou, *Structure theorems for complemented topological algebras*, Boll. U.M.I. **7**(1993), 961–971. MR1255657 (94k:46091). Zbl 0892.46053.

2010 Mathematics Subject Classification: 46H05; 46H10
Keywords: Fundamental complemented algebra, complemented linear space, fundamental complemented (topological) linear space, vector complementor, weakly fundamental algebra, axially closed element.

[5] M. Haralampidou, *Annihilator topological algebras*, Portug. Math. **51**(1994), 147–162. MR1281963 (95f:46076). Zbl 0806.46051.

[6] M. Haralampidou, *On complementing topological algebras*, J. Math. Sci., **96**(1999), No 6, 3722–3734. MR1724441 (2000j:46085). Zbl 0953.46024.

[7] M. Haralampidou, *Classification of locally m-convex algebras through Le Page condition*, Comment. Math. Prace Mat. **44**(2004), 255–269. MR2118012 (2005k: 46111). Zbl 1086.46035.

[8] M. Haralampidou, *Dual complementors in topological algebras*, Banach Center Publications, Institute of Math., Polish Academy of Sci. **67**(2005), 219–233. MR2143927 (2006e:46053). Zbl 1090.46036.

[9] M. Haralampidou, *On the Krull property in topological algebras*, Comment. Math. XLVI, **2**(2006), 141–162. MR2287681 (2007i:46045). Zbl 1180.46035.

[10] M. Haralampidou and K. Tzironis, *An application of the Kakutani-Mackey like theorem in the representation of complemented topological algebras* (in preparation)

[11] M. Haralampidou and K. Tzironis, *Representation theorems for fundamental complemented algebras* (in preparation)

[12] J. L. Kelley, *General Topology*, Springer-Verlag, New York, 1955. MR0070144 (16,1136c).

[13] A. Mallios, *Topological algebras. Selected topics*, North-Holland, Amsterdam, 1986. MR0857807 (87m:46099). Zbl 0597.46046.

[14] M. S. Moslehian, *A survey on the complemented subspace problem*, Trends in Math. **9**(2006), no. 1, 91–98.

[15] C. E. Rickart, *General theory of Banach algebras*, R.E. Krieger, Huntington, N.Y., 1974. MR0115101 (22 #5903). Zbl 095.09702.

[16] A. P. Robertson and W. Robertson, *Topological Vector Spaces*, Cambridge Univ. Press, 1964. MR0162118 (28 #5318). Zbl 0123.30202.

[17] B. J. Tomiuk, *Structure theory of complemented Banach algebras*, Can. J. Math. **14**(1962), 651–659. MR0143060 (26 #626).

[18] K. Tzironis, *On continuity of complementors in topological algebras*, Conference on Topological Algebras and their Applications (under publication, Proceedings of ICTAA 2014 - "De Gruyter Proceedings in Mathematics").

Marina Haralampidou
Department of Mathematics, University of Athens,
Panepistimioupolis, Athens 15784, Greece.
e-mail: mharalam@math.uoa.gr

Konstantinos Tzironis
Department of Mathematics, University of Athens,
Panepistimioupolis, Athens 15784, Greece.
e-mail: tzirk@math.uoa.gr

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