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On the Application of I. Vekua's Method for Geometrically Nonlinear and Non-Shallow Spherical Shells

Bakur Gulua

I. Vekua Institute of Applied Mathematics and Faculty of Exact and Natural Sciences of Iv. Javakhishvili Tbilisi State University, 2 University St., 0186, Tbilisi, Georgia; Sokhumi State Universit, 9 Anna Politkovskaia St., 0186, Tbilisi, Georgia

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In the present paper we consider the geometrically nonlinear and non-shallow spherical shells, when components of the deformation tensor have nonlinear terms. Using the method of I. Vekua and the method of a small parameter 2-D system of equations for the nonlinear and non-shallow spherical shells is obtained. Concrete problem has been solved.

Keywords: Stress vectors, Non-Shallow shells, Spherical shells, Small parameter.

AMS Subject Classification: 74K25.

There are many different methods of passage (reduction) from three- dimensional problems of elasticity to two-dimensional problems of the theory of shells. I.Vekua has obtained the equations of shallow shells [1],[2]. It means that the interior geometry of the shell does not vary in thickness. This method for non-shallow shells in case of geometrical and physical non-linear theory was generalized by T.Meunargia [3],[4].

A complete system of equations of the three-dimensional nonlinear theory of elasticity can be written as:

$$\partial_i\sqrt{g}\boldsymbol{\sigma}^i+\sqrt{g}\boldsymbol{\Phi}=0,\quad \Big(\partial_i=\frac{\partial}{\partial x^i}\Big),$$

$$\boldsymbol{\sigma}^i=\lambda\Big(\boldsymbol{R}^j\partial_j\boldsymbol{U}+\frac{1}{2}\partial^j\boldsymbol{U}\partial_j\boldsymbol{U}\Big)\Big(\boldsymbol{R}^i+\partial^i\boldsymbol{U})\Big)$$

$$+\mu\Big(\boldsymbol{R}^i\partial^j\boldsymbol{U}+\boldsymbol{R}^j\partial^i\boldsymbol{U}+\partial^i\boldsymbol{U}\partial^j\boldsymbol{U}\Big)\Big(\boldsymbol{R}_j+\partial_j\boldsymbol{U})\Big),$$

where x^1, x^2, x^3 are curvilinear coordinates, g is the discriminant of the metric tensor of the space, $\boldsymbol{\Phi}$ is an external force, $\boldsymbol{\sigma}^i$ are contravariant stress vectors,

*Corresponding author. Email: bak.gulua@gmail.com

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λ and μ are Lame's constants, $\boldsymbol{R}_i$ and $\boldsymbol{R}^i$ are covariant and contravariant base vectors of the space and $\boldsymbol{U}$ is the displacement vector

In the present paper we consider the system of equilibrium equations of the two-dimensional geometrically non-linear and non-shallow spherical shells which is obtained from the three-dimensional problems of the theory of elasticity for isotropic and homogeneous shells by the method of I. Vekua.

The displacement vector $\boldsymbol{U}(x^1, x^2, x^3)$ is expressed by the following formula [2]

$$\boldsymbol{U}(x^1, x^2, x^3) = \mathbf{u}(x^1, x^2) + \frac{x^3}{h}\mathbf{v}(x^1, x^2).$$

Here $\mathbf{u}(x^1, x^2)$ and $\mathbf{v}(x^1, x^2)$ are the vector fields on the middle surface $x^3 = 0$, $2h$ is the thickness of the shell, x^3 is a thickness coordinate $(-h \le x^3 \le h)$, x^1 and x^2 are isometric coordinates on the spherical surface.

The system of equilibrium equations of the two-dimensional geometrically non-linear and non-shallow spherical shells may be written in the following form (approximation $N = 1$):

$$\begin{aligned} &\partial_1 \overset{(0)}{\sigma}_{11} + \partial_2 \overset{(0)}{\sigma}_{21} + \varepsilon \overset{(0)}{\sigma}_{13} + \overset{(0)}{F}_1 = 0,\\ &\partial_1 \overset{(0)}{\sigma}_{12} + \partial_2 \overset{(0)}{\sigma}_{22} + +\varepsilon \overset{(0)}{\sigma}_{23} + \overset{(0)}{F}_2 = 0,\\ &\partial_1 \overset{(0)}{\sigma}_{13} + \partial_2 \overset{(0)}{\sigma}_{23} - \varepsilon \overset{(0)}{\sigma}_{11} + \overset{(0)}{F}_3 = 0, \end{aligned} \tag{1}$$

$$\begin{aligned} &\partial_1 \overset{(1)}{\sigma}_{11} + \partial_2 \overset{(1)}{\sigma}_{21} - \tfrac{3}{h} \overset{(0)}{\sigma}_{31} + \varepsilon \overset{(1)}{\sigma}_{13} + \overset{(1)}{F}_1 = 0,\\ &\partial_1 \overset{(1)}{\sigma}_{12} + \partial_2 \overset{(1)}{\sigma}_{22} - \tfrac{3}{h} \overset{(0)}{\sigma}_{32} + \varepsilon \overset{(1)}{\sigma}_{23} + \overset{(1)}{F}_2 = 0,\\ &\partial_1 \overset{(1)}{\sigma}_{13} + \partial_2 \overset{(1)}{\sigma}_{23} - 3 \overset{(0)}{\sigma}_{33} - \varepsilon \left(\overset{(1)}{\sigma}_{11} + \overset{(1)}{\sigma}_{22}\right) + \overset{(1)}{F}_3 = 0, \end{aligned} \tag{2}$$

where

$$\overset{(m)}{\boldsymbol{F}} = \overset{(m)}{\boldsymbol{\Phi}} + \frac{2m+1}{2h}\left[(1+\varepsilon)^2 \overset{(+)}{\boldsymbol{\sigma}}_3 - (-1)^m (1-\varepsilon)^2 \overset{(-)}{\boldsymbol{\sigma}}_3\right],$$

$$\left(\overset{(m)}{\sigma}_{ij},\ \overset{(m)}{\boldsymbol{\Phi}}\right) = \frac{2m+1}{2h}\int\limits_{-h}^{h}\left(1+\frac{x_3}{R}\right)^2 (\sigma_{ij},\ \boldsymbol{\Phi}) P_m\Big(\frac{x_3}{h}\Big) dx_3.$$

$$\boldsymbol{\sigma}_3(x_1, x_2, \pm h) = \overset{(\pm)}{\boldsymbol{\sigma}}_3.$$

Here P_m are Legendre polynomials of order m, $\varepsilon = \dfrac{h}{R_0}$ is a small parameter, R_0 is the radius of the middle surface of the sphere.

Let us construct the solutions of the form [5], [6]

$$u_i = \sum_{k=1}^{\infty} \overset{k}{u}_i \varepsilon^k, \qquad v_i = \sum_{k=1}^{\infty} \overset{k}{v}_i \varepsilon^k, \quad (i = 1, 2, 3), \tag{3}$$

where u_i and v_i are the components of the vectors $\mathbf{u}$ and $\mathbf{v}$ respectively.

Formal substitution of (3) into (2) and (1) shows that the series (3) may satisfy equations (1), (2) if the following equations are fulfilled [4]:

$$4\mu h^2 \frac{\partial}{\partial \bar{z}} \left(\frac{1}{\Lambda} \frac{\partial \overset{k}{u}_+}{\partial \bar{z}} \right) + 2(\lambda + \mu) h^2 \frac{\partial \theta^k}{\partial \bar{z}} + 2\lambda h \frac{\partial \overset{k}{v}_+}{\partial \bar{z}} = \overset{k}{X}_+, \tag{4}$$

$$\mu h^2 \nabla^2 \overset{k}{v}_3 - 3 \left[\lambda \overset{k}{\theta} + (\lambda + 2\mu) \overset{k}{v}_3 \right] = \overset{k}{X}_3,$$

$$4\mu h^2 \frac{\partial}{\partial \bar{z}} \left(\frac{1}{\Lambda} \frac{\partial \overset{k}{v}_+}{\partial \bar{z}} \right) + 2(\lambda + \mu) h^2 \frac{\partial \overset{k}{\Theta}}{\partial \bar{z}} - 3\mu \left(2h \frac{\partial \overset{k}{v}_3}{\partial \bar{z}} + \overset{k}{v}_+ \right) = \overset{k}{Y}_+, \tag{5}$$

$$\mu h \left(\nabla^2 \overset{k}{u}_3 + \overset{k}{\Theta} \right) = \overset{k}{Y}_3,$$

$$(k = 1, 2, ...),$$

where $z = x^1 + ix^2, \Lambda = \dfrac{4R_0^2}{(1 + z\bar{z})^2}, \nabla^2 = \dfrac{4}{\Lambda} \dfrac{\partial^2}{\partial z \partial \bar{z}}$ and

$$\overset{k}{u}_+ = \overset{k}{u}_1 + i \overset{k}{u}_2, \quad \overset{k}{v}_+ = \overset{k}{v}_1 + i \overset{k}{v}_2,$$

$$\overset{k}{\theta} = \frac{1}{\Lambda} \left(\frac{\partial \overset{k}{u}_+}{\partial z} + \frac{\partial \overset{k}{\overline{u}}_+}{\partial \bar{z}} \right), \quad \overset{k}{\Theta} = \frac{1}{\Lambda} \left(\frac{\partial \overset{k}{v}_+}{\partial z} + \frac{\partial \overset{k}{\overline{v}}_+}{\partial \bar{z}} \right).$$

Introducing the well-known differential operators

$$\frac{\partial}{\partial z} = \frac{1}{2} \left(\frac{\partial}{\partial x^1} - i \frac{\partial}{\partial x^2} \right), \quad \frac{\partial}{\partial \bar{z}} = \frac{1}{2} \left(\frac{\partial}{\partial x^1} + i \frac{\partial}{\partial x^2} \right).$$

$\overset{k}{X}_+, \overset{k}{Y}_+, \overset{k}{X}_3, \overset{k}{Y}_3$ are the components of external force and well-known quantity, defined by functions $\overset{0}{u}_i, ..., \overset{k-1}{u}_i, \overset{0}{v}_j, ..., \overset{k-1}{v}_j$.

The complex representation of general solutions of systems (4) end (5) are written in the following form

$$\overset{k}{u}_{+} = -\frac{5\lambda+6\mu}{3\lambda+2\mu}\frac{1}{\pi}\int\int_{D}\frac{\Lambda(\zeta,\bar{\zeta})\varphi'(\zeta)d\xi d\eta}{\bar{\zeta}-\bar{z}} + \left(\frac{1}{\pi}\int\int_{D}\frac{\Lambda(\zeta,\bar{\zeta})d\xi d\eta}{\bar{\zeta}-\bar{z}}\right)\overline{\varphi'(z)}$$
$$-\overline{\psi(z)} - \frac{\lambda h}{6(\lambda+\mu)}\frac{\partial\chi(z,\bar{z})}{\partial\bar{z}},$$
$$\overset{k}{v}_{3} = \chi(z,\bar{z}) - \frac{2\lambda h}{3\lambda+2\mu}\left(\varphi'(z)+\overline{\varphi'(z)}\right),$$
$$\overset{k}{v}_{+} = \frac{2(\lambda+2\mu)h^2}{3\mu}\overline{f''(z)} + \frac{1}{\pi}\int\int_{D}\frac{\Lambda(\zeta,\bar{\zeta})f'(\zeta)d\xi d\eta}{\bar{\zeta}-\bar{z}}$$
$$-\left(\frac{1}{\pi}\int\int_{D}\frac{\Lambda(\zeta,\bar{\zeta})d\xi d\eta}{\bar{\zeta}-\bar{z}}\right)\overline{f'(z)} - 2h\overline{g'(z)} + i\frac{\partial\omega(z,\bar{z})}{\partial\bar{z}},$$
$$\overset{k}{u}_{3} = g(z) + \overline{g(z)} - \frac{1}{\pi h}\int\int_{D}\Lambda(\zeta,\bar{\zeta})\left[f'(z)+\overline{f'(z)}\right]\ln|\zeta-z|d\xi d\eta,$$

where $\zeta=\xi+i\eta$, $\varphi(z)$,$\psi(z)$,$f(z)$ and $g(z)$ are arbitrary analytic functions of z, $\chi(z,\bar{z})$ and $\omega(z,\bar{z})$ are the general solutions of the following Helmholtz equations, respectively:

$$\Delta\chi - \kappa^2\chi = 0 \quad \left(\kappa^2 = \frac{3(\lambda+\mu)}{\lambda+2\mu}h^2\right),$$
$$\Delta\omega - \gamma^2\omega = 0 \quad \left(\gamma^2 = \frac{3}{h^2}\right).$$

D is the domain of the plane Ox^1x^2 onto which the midsurface S of the shell is mapped topologically.

Here we present a general scheme of solution of the boundary value problem when the domain D is a circle of radius r_0.

The boundary value problem (in displacements) for any k takes the form

$$\overset{k}{u}_{+}\Big|_{r_0} = \Big\{-\frac{5\lambda+6\mu}{3\lambda+2\mu}\frac{1}{\pi}\int\int_{D}\frac{\Lambda(\zeta,\bar{\zeta})\varphi'(\zeta)d\xi d\eta}{\bar{\zeta}-\bar{z}} + \left(\frac{1}{\pi}\int\int_{D}\frac{\Lambda(\zeta,\bar{\zeta})d\xi d\eta}{\bar{\zeta}-\bar{z}}\right)\overline{\varphi'(z)}$$
$$-\overline{\psi(z)} - \frac{\lambda h}{6(\lambda+\mu)}\frac{\partial\chi(z,\bar{z})}{\partial\bar{z}}\Big\}_{r_0} = \overset{k}{G}_{+}, \quad (|z|=r_0) \qquad (6)$$
$$\overset{k}{v}_{3}\Big|_{r_0} = \left\{\chi(z,\bar{z}) - \frac{2\lambda h}{3\lambda+2\mu}\left(\varphi'(z)+\overline{\varphi'(z)}\right)\right\}_{r_0} = \overset{k}{G}_{3}, \qquad (7)$$
$$\overset{k}{v}_{+}\Big|_{r_0} = \Big\{\frac{2(\lambda+2\mu)h^2}{3\mu}\overline{f''(z)} + \frac{1}{\pi}\int\int_{D}\frac{\Lambda(\zeta,\bar{\zeta})f'(\zeta)d\xi d\eta}{\bar{\zeta}-\bar{z}}$$

$$-\left(\frac{1}{\pi}\int\int\limits_{D}\frac{\Lambda(\zeta,\bar{\zeta})d\xi d\eta}{\bar{\zeta}-\bar{z}}\right)\overline{f'(z)}-2h\overline{g'(z)}+i\frac{\partial\omega(z,\bar{z})}{\partial\bar{z}}\Big\}_{r_0}=\overset{k}{J}_{+},\ (|z|=r_0),\ (8)$$

$$\overset{k}{u}{}_3\Big|_{r_0}=\Big\{g(z)+\overline{g(z)}-\frac{1}{\pi h}\int\int\limits_{D}\Lambda(\zeta,\bar{\zeta})\left[f'(z)+\overline{f'(z)}\right]\ln|\zeta-z|d\xi d\eta\Big\}_{r_0}=\overset{k}{J}_3\ (9)$$

$$(k=0,1,...\ z=re^{i\vartheta},\ \ \zeta=\rho e^{i\vartheta}),$$

where $\overset{k}{G}_{+}$, $\overset{k}{G}_3$, $\overset{k}{J}_{+}$, $\overset{k}{J}_3$ are the known values.

Let us introduce the functions $\varphi'(z)$, $\psi(z)$ and $\chi(z,\bar{z})$ by the series

$$\varphi'(z)=\sum_{n=0}^{\infty}a_nz^n,\quad \psi(z)=\sum_{n=0}^{\infty}b_nz^n,\quad \chi(z,\bar{z})=\sum_{-\infty}^{\infty}\alpha_nI_n(\kappa r)e^{in\vartheta},\qquad (10)$$

$$\overset{k}{G}_{+}=\sum_{-\infty}^{\infty}A_ne^{in\vartheta},\quad \overset{k}{G}_3=\sum_{-\infty}^{\infty}B_ne^{in\vartheta},\qquad (11)$$

where $I_n(\kappa r)$ are Bessel's modificed functions.

By substituting (10), (11) into (6) and (7) we obtain the system of algebraic equations:

$$\frac{5\lambda+6\mu}{3\lambda+2\mu}\frac{\delta_0}{r_0}a_0-\frac{\delta_0}{r_0}\bar{a}_0-\frac{\lambda\kappa h}{12(\lambda+\mu)}I_1(\kappa r_0)\alpha_0=A_1,$$

$$I_0(\kappa r_0)\alpha_0-\frac{2\lambda h}{3\lambda+2\mu}(a_0+\bar{a}_0)=B_0,$$

$$\frac{5\lambda+6\mu}{3\lambda+2\mu}\frac{\delta_{n-1}}{r_0^n}a_{n-1}-\frac{\lambda\kappa h}{12(\lambda+\mu)}I_n(\kappa r_0)\alpha_{n-1}=A_n,\ \ (n\geq 2),$$

$$\delta_0r_0^n\bar{a}_{n+1}+r_0^n\bar{b}_n-\frac{\lambda\kappa h}{12(\lambda+\mu)}I_n(\kappa r_0)\alpha_{-n-1}=-A_{-n},\ \ (n\geq 0),$$

$$I_n(\kappa r_0)\alpha_n-\frac{2\lambda h}{3\lambda+2\mu}r_0^na_n=B_n,\ \ (n\geq 1).$$

For coefficients a_n, b_n and α_n we have:

$$a_n=\frac{(3\lambda+2\mu)\left(A_{n+1}+\dfrac{\lambda\kappa hI_{n+1}(\kappa r_0)}{12(\lambda+\mu)I_n(\kappa r_0)}B_n\right)}{(5\lambda+6\mu)\dfrac{\delta_{n-1}}{r_0^n}-\dfrac{\lambda^2h^2\kappa r_0^nI_{n+1}(\kappa r_0)}{6(\lambda+\mu)I_n(\kappa r_0)}},\ \ (n\geq 1)$$

$$\alpha_n=\frac{1}{I_n(\kappa r_0)}\left(B_n+\frac{2\lambda hr_0^n\left(A_{n+1}+\dfrac{\lambda\kappa hI_{n+1}(\kappa r_0)}{12(\lambda+\mu)I_n(\kappa r_0)}B_n\right)}{(5\lambda+6\mu)\dfrac{\delta_{n-1}}{r_0^n}-\dfrac{\lambda^2h^2\kappa r_0^nI_{n+1}(\kappa r_0)}{6(\lambda+\mu)I_n(\kappa r_0)}}\right),\ \ (n\geq 1)$$

$$b_n = -\frac{\bar{A}_{-n}}{r_0^n} - \frac{(3\lambda+2\mu)\delta_0\left(A_{n+2}+\dfrac{\lambda\kappa h I_{n+2}(\kappa r_0)}{12(\lambda+\mu)I_{n+1}(\kappa r_0)}B_{n+1}\right)}{(5\lambda+6\mu)\dfrac{\delta_n}{r_0^{n+1}}-\dfrac{\lambda^2h^2\kappa r_0^{n+1}I_{n+2}(\kappa r_0)}{6(\lambda+\mu)I_{n+1}(\kappa r_0)}}$$

$$-\frac{\lambda\kappa h I_n(\kappa r_0)}{12(\lambda+\mu)r_0^n I_{n+1}(\kappa r_0)}$$

$$\times\left(B_{n+1}+\frac{2\lambda h r_0^{n+1}\left(A_{n+2}+\dfrac{\lambda\kappa h I_{n+2}(\kappa r_0)}{12(\lambda+\mu)I_{n+1}(\kappa r_0)}B_{n+1}\right)}{(5\lambda+6\mu)\dfrac{\delta_n}{r_0^{n+1}}-\dfrac{\lambda^2h^2\kappa r_0^{n+1}I_{n+2}(\kappa r_0)}{6(\lambda+\mu)I_{n+1}(\kappa r_0)}}\right),\quad (n\geq 0)$$

$$a_0 = \frac{(3\lambda+2\mu)\left(ReA_1+\dfrac{\lambda\kappa h I_1(\kappa r_0)}{12(\lambda+\mu)I_0(\kappa r_0)}B_0\right)}{2(\lambda+2\mu)\dfrac{\delta_0}{r_0}-\dfrac{\lambda^2h^2\kappa r_0 I_1(\kappa r_0)}{3(\lambda+\mu)I_0(\kappa r_0)}}+i\frac{3\lambda+2\mu}{8(\lambda+\mu)}\frac{r_0}{\delta_0}ImA_1,$$

$$\alpha_0 = \frac{B_0}{I_0(\kappa r_0)}+\frac{4\lambda h\left(ReA_1+\dfrac{\lambda\kappa h I_1(\kappa r_0)}{12(\lambda+\mu)I_0(\kappa r_0)}B_0\right)}{2(\lambda+2\mu)\delta_0 I_0(\kappa r_0)-\dfrac{\lambda^2h^2\kappa r_0^2 I_1(\kappa r_0)}{3(\lambda+\mu)}},$$

where $\delta_n = 8R^2\displaystyle\int_0^{r_0}\frac{\rho^{2n+1}}{(1+\rho^2)^2}d\rho.$

Let us introduce the functions $f'(z),\ g(z)$and $\omega(z,\bar{z})$ by the series

$$f'(z)=\sum_{n=0}^{\infty}c_nz^n,\quad g(z)=\sum_{n=0}^{\infty}d_nz^n,\quad \omega(z,\bar{z})=\sum_{-\infty}^{\infty}\beta_nI_n(\gamma r)e^{in\theta}. \tag{12}$$

$$\overset{k}{J}_+=\sum_{-\infty}^{\infty}M_ne^{in\vartheta},\quad \overset{k}{J}_3=\sum_{-\infty}^{\infty}N_ne^{in\vartheta}. \tag{13}$$

We now find the coefficients c_n, d_n, and β_n from the following system of algebraic equations:

$$\frac{i\gamma}{2}I_1(\gamma r_0)\beta_0+\frac{\delta_0}{r_0}(c_0+\bar{c}_0)=M_1,$$

$$d_0+\bar{d}_0-\frac{\delta_0}{h}(c_0+\bar{c}_0)=N_0,$$

$$\frac{i\gamma}{2}I_n(\gamma r_0)\beta_{n-1}+\frac{\delta_{n-1}}{r_0^n}c_{n-1}=M_n,\quad (n\geq 2),$$

$$\frac{i\gamma}{2}I_n(\gamma r_0)\beta_{-n-1}-2h(n+1)r_0^n\bar{d}_{n+1}$$
$$+\left(\delta_0+\frac{2(\lambda+2\mu)h^2}{3\mu}(n+1)\right)r_0^n\bar{c}_{n+1}=M_{-n},\ \ (n\geq 0),$$
$$r_0^nd_n-\frac{\delta_n}{nr_0^nh}c_n=N_n,\ \ (n\geq 1).$$

The solutions of the system have the following forms:

$$c_{n+1}=\frac{\overline{M}_{-n}+\dfrac{I_n(\gamma r_0)}{I_{n+2}(\gamma r_0)}M_{n+2}+\dfrac{2h(n+1)}{r_0}N_{n+1}}{\left(\dfrac{I_n(\gamma r_0)}{I_{n+2}(\gamma r_0)}-2\right)\dfrac{\delta_{n+1}}{r_0^{n+2}}+\delta_0r_0^n+\dfrac{2(\lambda+2\mu)(n+1)h^2r_0^n}{3\mu}},\ \ (n\geq 0)$$

$$d_n=\frac{1}{r_0^n}\left(N_n+\frac{\delta_n}{nr_0^nh}\frac{\overline{M}_{-n+1}+\dfrac{I_{n-1}(\gamma r_0)}{I_{n+1}(\gamma r_0)}M_{n+1}+\dfrac{2hn}{r_0}N_n}{\left(\dfrac{I_{n-1}(\gamma r_0)}{I_{n+1}(\gamma r_0)}-2\right)\dfrac{\delta_n}{r_0^{n+1}}+\delta_0r_0^{n-1}+\dfrac{2(\lambda+2\mu)nh^2r_0^{n-1}}{3\mu}}\right),$$

$$\beta_n=\frac{2i}{\gamma I_{n+1}(\gamma r_0)}$$
$$\times\left(M_{n+1}-\frac{\delta_n}{r_0^{n+1}}\frac{\overline{M}_{-n+1}+\dfrac{I_{n-1}(\gamma r_0)}{I_{n+1}(\gamma r_0)}M_{n+1}+\dfrac{2hn}{r_0}N_n}{\left(\dfrac{I_{n-1}(\gamma r_0)}{I_{n+1}(\gamma r_0)}-2\right)\dfrac{\delta_n}{r_0^{n+1}}+\delta_0r_0^{n-1}+\dfrac{2(\lambda+2\mu)nh^2r_0^{n-1}}{3\mu}}\right),$$

$$c_0+\bar{c}_0=\frac{ReA_1r_0}{\delta_0},\ \ d_0+\bar{d}_0=N_0+\frac{ReA_1r_0}{h},\ \ \beta_0=\frac{2ImA_1}{\gamma I_1(\gamma r_0)}.$$

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