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PARTS OF REDUCING SYSTEMS OF PARAMETERS

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Abstract. In the present paper we want to give some connections between parts of reducing systems of parameters and the Cohen–Macaulay-property. All proofs are given in [**4**].

Let R be a local ring with maximal ideal $\mathfrak{m}$, and let $M \neq 0$ be a f.g. R-module of dimension d.

Definition. A system of parameters $(x_1, \dots, x_d)$ of M is called *reducing* if for all $i = 1, \dots, d-1$ we have

$$x_i \notin P \text{ for all } P \in \operatorname{Ass} M/(x_1, \dots, x_{i-1})M \text{ with } \dim R/P = d - i.$$

A sequence $x_1, \dots, x_r$ of elements of $\mathfrak{m}$ is called *part of a (reducing) system of parameters* of M if there are elements $x_{r+1}, \dots, x_d \in \mathfrak{m}$ such that $(x_1, \dots, x_r, x_{r+1}, \dots, x_d)$ is a (reducing) system of parameters of M.

In [**1**] it is shown that for every system of parameters $(x_1, \dots, x_d)$ there is a reducing system of parameters $(y_1, \dots, y_d)$, such that

$$(x_1, \dots, x_d)M = (y_1, \dots, y_d)M.$$

This definition is equivalent to that given in [**1**] (shown in [**4**]).

Remark. If M is Cohen–Macaulay, then the concepts of a regular sequence, a part of a reducing system of parameters and a part of a system of parameters coincide.

Theorem. *Given a reducing system of parameters $(y_1, \dots, y_d)$, M is Cohen–Macaulay iff y_d is a non zerodivisor on $M/(y_1, \dots, y_{d-1})M$.*

The proof is given in [**4**] using the following Lemma.

Lemma. *If $x \in \mathfrak{m}_R$ is a zerodivisor on M, then $P \in \operatorname{Ass} M/xM$ for all minimal primes $P \in \operatorname{Ass} M \cap \operatorname{V}(x)$.*

The main result determines connetions between parts of reducing systems of parameters and the Cohen–Macaulay-property:

Theorem. *Let $(x_1, \ldots, x_r)$ be part of a system of parameters of M, where $0 \leq r < d$. Then the following conditions are equivalent:*

(i) *$(x_1, \ldots, x_r)$ is part of a reducing system of parameters of M.*

(ii) *M_P is an r-dimensional Cohen–Macaulay module over R_P for all $P \in$* $\operatorname{Supp} M \cap \operatorname{V}(x_1, \ldots, x_r)$ *satisfying* $\dim R/P = \dim M - r$.

(iii) *There is a part $(y_1, \ldots, y_r)$ of a reducing system of parameters of M, such that $(y_1, \ldots, y_r)M = (x_1, \ldots, x_r)M$.*

(iv) *There is a part $(y_1, \ldots, y_r)$ of a reducing system of parameters of M, such that* $\operatorname{Supp} M \cap \operatorname{V}(x_1, \ldots, x_r) \subseteq V(y_1, \ldots, y_r)$.

As a first conclusion we note that any permutation of a part of a reducing system of parameters is again a part of a reducing system of parameters. This is impossible in the non reducing case.

Finally, we define the *strong Cohen–Macaulay locus* of $\operatorname{Supp} M$ by

$$\mathcal{CM}(M) := \{P \in \operatorname{Supp} M \mid \dim R/P + \dim M_P = d \text{ and } M_P \text{ is a Cohen–Macaulay module over } R_P\}$$

and for $0 \leq r \leq d$ we set

$$\mathcal{CM}_r(M) := \{P \in \mathcal{CM}(M) \mid \dim M_P = r\}.$$

Proposition. *For $r \in \mathbb{N}$, $r < d$ we have*

$$\mathcal{CM}_r(M) = \{P \mid P \in \operatorname{Ass} M/(x_1, \ldots, x_r)M,\ \dim R/P = d - r,\ (x_1, \ldots, x_r) \textit{ is a part of a reducing system of parameters of } M\}.$$

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