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Twisted Alexander polynomials and surjectivity of a group homomorphism

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Abstract If $\varphi\colon G \to G'$ is a surjective homomorphism, we prove that the twisted Alexander polynomial of G is divisible by the twisted Alexander polynomial of G'. As an application, we show non-existence of surjective homomorphism between certain knot groups.

AMS Classification 57M25; 57M05

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1 Introduction

Suppose that G is a finitely presentable group with a surjective homomorphism to the free abelian group of rank l, eg, abelianization. Let $\rho\colon G \to GL(n;R)$ be a linear representation. The twisted Alexander polynomial of G associated to ρ was introduced in [10] and is defined to be a rational expression of l indeterminates.

Let $\varphi\colon G \to G'$ be a surjective homomorphism. Each representation $\rho'\colon G' \to GL(n;R)$ naturally induces a representation of G, namely, $\rho = \rho' \circ \varphi$. In this paper we prove the following:

Main theorem *The twisted Alexander polynomial of G associated to ρ is divisible by the twisted Alexander polynomial of G' associated to ρ'.*

The corresponding fact about the Alexander polynomial is known [1].

We present two separate proofs of the main theorem. First we give a purely algebraic proof in §3. If G is a knot group, the twisted Alexander polynomial of G may be regarded as the Reidemeister torsion. In §4, we provide another proof of the main theorem in case when G and G' are knot groups, from the view point of the Reidemeister torsion.

In the last section, we show non-existence of surjective homomorphism between certain knot groups, as an application of the main theorem.

2 Twisted Alexander polynomial

In this section, we recall briefly the definition of the twisted Alexander polynomial.

Let G be a finitely presentable group. Choose and fix a presentation as follows:

$$G = \langle x_1, \dots, x_u \mid r_1, \dots, r_v \rangle.$$

We denote by $\alpha\colon G \to \mathbb{Z}^l$ a surjective homomorphism to the free abelian group with generators $t_1, \dots, t_l$ and $\rho\colon G \to GL(n; R)$ a linear representation, where R is a unique factorization domain. These maps naturally induce ring homomorphisms $\tilde{\rho}$ and $\tilde{\alpha}$ from $\mathbb{Z}[G]$ to $M(n; R)$ and $\mathbb{Z}[{t_1}^{\pm1}, \dots, {t_l}^{\pm1}]$ respectively, where $M(n; R)$ denotes the matrix algebra of degree n over R. Then $\tilde{\rho} \otimes \tilde{\alpha}$ defines a ring homomorphism

$$\mathbb{Z}[G] \to M\left(n; R[{t_1}^{\pm1}, \dots, {t_l}^{\pm1}]\right).$$

Let F_u be the free group on generators $x_1, \dots, x_u$ and

$$\Phi\colon\ \mathbb{Z}[F_u] \to M\left(n; R[{t_1}^{\pm1}, \dots, {t_l}^{\pm1}]\right)$$

the composite of the surjection $\mathbb{Z}[F_u] \to \mathbb{Z}[G]$ induced by the fixed presentation and the map $\tilde{\rho} \otimes \tilde{\alpha}\colon\ \mathbb{Z}[G] \to M(n; R[{t_1}^{\pm1}, \dots, {t_l}^{\pm1}])$.

We define the $v \times u$ matrix M whose (i, j) component is the $n \times n$ matrix

$$\Phi\left(\frac{\partial r_i}{\partial x_j}\right) \in M\left(n; R[{t_1}^{\pm1}, \dots, {t_l}^{\pm1}]\right),$$

where $\partial/\partial x$ denotes the Fox derivation. This matrix M is called the Alexander matrix of the presentation of G associated to the representation ρ.

It is easy to see that there is an integer $1 \le j \le u$ such that $\det \Phi(x_j - 1) \ne 0$. For such j, let us denote by M_j the $v \times (u-1)$ matrix obtained from M by removing the j-th column. We regard M_j as an $nv \times n(u-1)$ matrix with coefficients in $R[{t_1}^{\pm1}, \dots, {t_l}^{\pm1}]$. Moreover, for an $n(u-1)$-tuple of indices

$$I = \left(i_1, i_2, \dots, i_{n(u-1)}\right), \quad \left(1 \le i_1 < i_2 < \cdots < i_{n(u-1)} \le nv\right)$$

we denote by M_j^I the $n(u-1)\times n(u-1)$ square matrix consisting of the i_k-th row of the matrix M_j, where $k=1,2,\dots,n(u-1)$.

Then the twisted Alexander polynomial (see [10]) of a finitely presented group G for a representation $\rho\colon G\to GL(n;R)$ is defined to be a rational expression

$$\Delta_{G,\rho}(t_1,\dots,t_l)=\frac{\gcd_I(\det M_j^I)}{\det\Phi(x_j-1)}$$

and moreover is well-defined up to a factor $\epsilon {t_1}^{\varepsilon_1}\cdots {t_l}^{\varepsilon_l}$, where $\epsilon\in R^\times, \varepsilon_i\in\mathbb{Z}$. See [10], [7], [2] and [3] for more precise definition and applications.

3 Main theorem and the algebraic proof

In this section, we prove the following main theorem of this paper.

Theorem 3.1 *Let G and G' be finitely presentable groups and α,α' surjective homomorphisms from G,G' to $\mathbb{Z}^l$ respectively. Suppose that there exists a surjective homomorphism $\varphi\colon G\to G'$ such that $\alpha=\alpha'\circ\varphi$. Then $\Delta_{G,\rho}$ is divisible by $\Delta_{G',\rho'}$ for any representation $\rho'\colon G'\to GL(n;R)$, where $\rho=\rho'\circ\varphi$. That is to say, the quotient of $\Delta_{G,\rho}$ by $\Delta_{G',\rho'}$ is a genuine polynomial.*

Proof Choose and fix a presentation

$$G=\langle x_1,x_2,\dots,x_u \mid r_1,r_2,\dots,r_v\rangle.$$

Since φ is surjective, then G' is generated by $\varphi(x_1),\dots,\varphi(x_u)$. Namely, G' can be presented as

$$G'=\langle \varphi(x_1),\varphi(x_2),\dots,\varphi(x_u) \mid s_1,s_2,\dots,s_{v'}\rangle.$$

For convenience, we also write x_i for $\varphi(x_i)$, that is, we consider that G' is generated by $x_1,\dots,x_u$. By this notation, each relator r_i is written as

$$r_i=\prod_k u_k s_{l_{i_k}}^{\varepsilon_{i_k}} u_k^{-1},\quad i=1,2,\dots,v,\ 1\le l_{i_k}\le v',\ u_k\in F_u,\ \varepsilon_{i_k}=\pm 1,$$

since φ is a homomorphism. By applying the Fox derivation $\frac{\partial}{\partial x_j}$ and collecting terms of $\frac{\partial s_k}{\partial x_j}$, we get

$$\varphi\left(\frac{\partial r_i}{\partial x_j}\right)=\sum_{k=1}^{v'} A_{i,k}\,\frac{\partial s_k}{\partial x_j}. \tag{1}$$

Here $A_{i,k}$ $(1 \leq i \leq v)$ is a sum of some $\varepsilon_\bullet\,\varphi(u_\bullet)$, which does not depend on j. Let M_G and $M_{G'}$ be the Alexander matrices with the u-th column removed:

$$M_G = \begin{pmatrix} \tilde\rho\otimes\tilde\alpha\left(\frac{\partial r_1}{\partial x_1}\right) & \cdots & \tilde\rho\otimes\tilde\alpha\left(\frac{\partial r_1}{\partial x_{u-1}}\right) \\ \vdots & \ddots & \vdots \\ \tilde\rho\otimes\tilde\alpha\left(\frac{\partial r_v}{\partial x_1}\right) & \cdots & \tilde\rho\otimes\tilde\alpha\left(\frac{\partial r_v}{\partial x_{u-1}}\right) \end{pmatrix}$$

$$M_{G'} = \begin{pmatrix} \tilde\rho'\otimes\tilde\alpha'\left(\frac{\partial s_1}{\partial x_1}\right) & \cdots & \tilde\rho'\otimes\tilde\alpha'\left(\frac{\partial s_1}{\partial x_{u-1}}\right) \\ \vdots & \ddots & \vdots \\ \tilde\rho'\otimes\tilde\alpha'\left(\frac{\partial s_{v'}}{\partial x_1}\right) & \cdots & \tilde\rho'\otimes\tilde\alpha'\left(\frac{\partial s_{v'}}{\partial x_{u-1}}\right) \end{pmatrix}.$$

By (1), we have

$$M_G = AM_{G'}$$

where $A = (\rho'(A_{i,k}))$ is a $nv \times nv'$ matrix. For $I = (i_1, i_2, \dots, i_{n(u-1)})$, as is easily shown,

$$\det M_G^I = \det\left(A^I M_{G'}\right) = \sum_K \pm\left(\det A_K^I\right)\left(\det M_{G'}^K\right)$$

where $K = (k_1, k_2, \dots, k_{n(u-1)})$ and A_K^I is the matrix consisting of the $k_1, k_2, \dots, k_{n(u-1)}$-th columns of A^I. It follows that if $\det M_{G'}^I$ has a common divisor P for all I, then so does $\det M_G^I$. Moreover, the denominator of $\Delta_{G,\rho}$ is equal to that of $\Delta_{G',\rho'}$. This completes the proof. □

The corresponding fact about the Alexander polynomial is well known. Let $G(K)$ be the knot group $\pi_1(S^3 - K)$ of a knot K in S^3. For any knots K, K', if there exists a surjective homomorphism from $G(K)$ to $G(K')$, then the Alexander polynomial of K is divisible by that of K'. Murasugi mentions that if there exists a surjective homomorphism from a knot group $G(K)$ to the trefoil knot group, then the twisted Alexander polynomial of $G(K)$ is divisible by that of the trefoil knot group. The main theorem is a generalization of the above.

We will now make a few remarks about geometric settings in which surjective homomorphisms arise. First we consider the case of degree one maps. Let X and Y be d-dimensional compact manifolds. Suppose that $f\colon X \to Y$ is a degree one map. It is easy to see that its induced homomorphism $f_*\colon \pi_1(X) \to \pi_1(Y)$ is a surjective homomorphism.

In the knot group case, there exist the following situations except for degree 1 maps. First, there exists a surjective homomorphism from any knot group to the trivial knot group which is the infinite cyclic group. Secondly, if a knot K is a connected sum of K_1 and K_2, then its knot group $G(K)$ is an amalgamated product of $G(K_1)$ and $G(K_2)$. Then there exists a surjection from $G(K)$ to each factor group. Thirdly, if a knot K is a periodic knot of order n, then there exists a surjective homomorphism from $G(K)$ to $G(K_*)$ where K_* is its quotient knot of K.

4 Another proof from the view point of the Reidemeister torsion

In this section, we prove our theorem in the knot group case. It is done by using the Mayer-Vietoris argument of the Reidemeister torsion.

Here let us consider a knot K in S^3 and its exterior $E(K)$. For the knot group $G(K) = \pi_1 E(K)$, we choose and fix a Wirtinger presentation

$$G(K) = \langle x_1, \dots, x_u \mid r_1, \dots, r_{u-1} \rangle.$$

The abelianization homomorphism

$$\alpha_K\colon\ G(K) \to H_1(E(K), \mathbb{Z}) \cong \mathbb{Z} = \langle t \rangle$$

is given by $\alpha_K(x_1) = \cdots = \alpha_K(x_u) = t$. If we have no confusion, we write simply α for α_K as in the previous section. In this section, we take a unimodular representation $\rho\colon\ G(K) \to SL(n; \mathbb{F})$ over a field $\mathbb{F}$. As in the definition of the twisted Alexander polynomial, we consider the tensor representation

$$\rho \otimes \alpha\colon\ G \to GL(n; \mathbb{F}[t, t^{-1}]) \subset GL(n; \mathbb{F}(t)).$$

Here $\mathbb{F}(t)$ denotes the rational function field over $\mathbb{F}$. If $\rho \otimes \alpha$ is an acyclic representation over $\mathbb{F}(t)$, that is, all homology groups over $\mathbb{F}(t)$ of $E(K)$ twisted by $\rho \otimes \alpha$ are vanishing, then the Reidemeister torsion of $E(K)$ for $\rho \otimes \alpha$ can be defined. Furthermore the following equality holds. See [3, 4] for more details of definitions and proofs.

Theorem 4.1 *If $\rho \otimes \alpha$ is an acyclic representation, then we have*

$$\tau_{\rho \otimes \alpha}(E(K)) = \Delta_{G(K),\rho}(t)$$

up to a factor $\pm t^{nk}$ ($k \in \mathbb{Z}$) if n is odd, and up to only t^{nk} if n is even.

From this theorem, we prove the main theorem as divisibility of the Reidemeister torsion in the knot group case. Here we take a surjective homomorphism

$\varphi\colon G(K) \to G(K')$. By changing the orientation of meridians if we need, we may assume that $\alpha_{K'} \circ \varphi = \alpha_K$. Let $\rho'\colon G(K') \to SL(n;\mathbb{F})$ be a representation. For simplicity, we write the composition $\rho = \rho' \circ \varphi$.

Now we consider 2-dimensional CW-complexes $X(K)$ and $X(K')$ defined by their Wirtinger presentations. It is well-known that these complexes are simple homotopy equivalent to the knot exteriors. Then these Reidemeister torsions of $X(K)$ and $X(K')$ are equal to the twisted Alexander polynomials respectively. Here we consider twisted homologies of these complexes by using their CW-complex structure. The coefficient V is a $2n$-dimensional vector space over a rational function field $\mathbb{F}(t)$. When V is regarded as a $G(K)$-module by using ρ, it is denoted by V_ρ.

The surjective homomorphism φ induces a chain map $\varphi_*\colon C_*(X(K), V_\rho) \to C_*(X(K'), V_{\rho'})$. We take a tensor representation $\rho \otimes \alpha_K\colon G(K) \to GL(n;\mathbb{F}(t))$. Assume that $\rho \otimes \alpha_K$ and $\rho' \otimes \alpha_{K'}$ are acyclic representations. Then we can prove the following.

Theorem 4.2 *The quotient $\tau(X(K);V_{\rho\otimes\alpha_K})/\tau(X(K');V_{\rho'\otimes\alpha_{K'}})$ is a polynomial in $\mathbb{F}[t,t^{-1}]$.*

We show the following proposition first.

Proposition 4.3 *The chain map*

$$\varphi_*\colon C_*(X(K), V_{\rho\otimes\alpha_K}) \to C_*(X(K'), V_{\rho'\otimes\alpha_{K'}})$$

is surjective.

Proof It is clear that φ induces an isomorphism on the 0-chains, and a surjection on the 1-chains. Then we only need to prove this proposition on the 2-chains.

We take a non-trivial 2-chain $z \in C_2(X(K'), V_{\rho'\otimes\alpha_{K'}})$. By the acyclicity of the chain complex $C_*(X(K'), V_{\rho'\otimes\alpha_{K'}})$, the boundary map $\partial\colon C_2(X(K'), V_{\rho'\otimes\alpha_{K'}}) \to C_1(X(K'), V_{\rho'\otimes\alpha_{K'}})$ is injective. Then the image ∂z is non-trivial in C_1. On the other hand, by the surjectivity of

$$\varphi\colon C_1(X(K), V_{\rho\otimes\alpha_K}) \to C_1(X(K'), V_{\rho'\otimes\alpha_{K'}}),$$

there exists a 2-chain $w \in C_2(X(K), V_{\rho\otimes\alpha_K})$ such that $\varphi_*(w) = z$. By the commutativity of maps, in C_2

$$\varphi_*(\partial w) = \partial\varphi_*(w) = \partial\partial z = 0.$$

Then we have $\partial w = 0$. By the acyclicity, there exists $\tilde{w} \in C_*(X(K), V_{\rho\otimes\alpha_K})$ such that $\partial\tilde{w} = w$. Again by the commutativity, $\varphi(\tilde{w}) = z$. Therefore φ_* is surjective. □

Proof of Theorem 4.2 From the above proposition, we can take the kernel D_* of this chain map φ_* and obtain a short exact sequence

$$0 \to D_* \to C_*(X(K), V_{\rho\otimes\alpha_K}) \to C_*(X(K'), V_{\rho'\otimes\alpha_{K'}}) \to 0.$$

Here we recall the following fact. For a short exact sequence $0 \to C'_* \to C_* \to C''_* \to 0$ of finite chain complexes, if two of them are acyclic complexes, then the third one is also acyclic. Furthermore, the torsion satisfies

$$\tau(C_*) = \tau(C'_*)\tau(C''_*)$$

up to some factor.

By applying the property of the product of torsion, we have

$$\tau(X(K); V_{\rho\otimes\alpha_K}) = \tau(X(K'); V_{\rho'\otimes\alpha_{K'}})\tau(D; V_{\rho\otimes\alpha_K}).$$

We only need to prove that $\tau(D; V_{\rho\otimes\alpha_K})$ is a polynomial. From the definition we see that D_0 vanishes, since

$$\varphi_*\colon\ C_0(X(K), V_{\rho\otimes\alpha_K}) \to C_0(X(K'), V_{\rho'\otimes\alpha_{K'}})$$

is isomorphism. Hence by definition, its torsion is the determinant of $D_2 \to D_1$. Therefore it is a polynomial. □

Remark 4.4 By a similar argument, we can prove that if $\varphi\colon G(K) \to G(K')$ is an injective homomorphism, then $\tau(X(K'); V_{\rho\otimes\alpha_{K'}})/\tau(X(K); V_{\rho\otimes\alpha_K})$ is a polynomial.

5 Examples

In this section, we show some examples of the twisted Alexander polynomials and an application of Theorem 3.1. We consider the problem: Is there a surjective homomorphism from $G(K)$ to $G(K')$ for two given knots K, K'? The problem has been investigated by Murasugi when K' is the trefoil knot 3_1 (c.f. [8]). Here we study the problem in case when K' is the figure eight knot 4_1. The numbering of the knots follows that of Rolfsen's book [9].

If the classical Alexander polynomial of K can not be divided by that of K', we know that there are no surjective homomorphisms from $G(K)$ to $G(K')$.

In the knot table in [9], up to 9 crossings, the classical Alexander polynomial of each knot is not divisible by that of $G(4_1)$ except for $8_{18}, 8_{21}, 9_{12}, 9_{24}, 9_{37}$, 9_{39} and 9_{40}. That is to say, except for $8_{18}, 8_{21}, 9_{12}, 9_{24}, 9_{37}$, 9_{39} and 9_{40}, there exists no surjective homomorphisms from such a knot group to $G(4_1)$.

Next, we consider a representation $\rho\colon G(K) \to SL(2;\mathbb{Z}/p\mathbb{Z})$ and the twisted Alexander polynomial associated to ρ. Theorem 3.1 says that if the numerator of $\Delta_{G(K),\rho}$ for all representations $\rho\colon G(K) \to SL(2;\mathbb{Z}/p\mathbb{Z})$ for some fixed prime p cannot be divided by the numerator of $\Delta_{G(K'),\rho'}$ for a certain representation $\rho'\colon G(K') \to SL(2;\mathbb{Z}/p\mathbb{Z})$, then there exists no surjective homomorphisms from $G(K)$ to $G(K')$.

Let us compute the twisted Alexander polynomials $\Delta_{G(4_1),\rho'}$ for a certain representation $\rho'\colon G(4_1) \to SL(2;\mathbb{Z}/7\mathbb{Z})$. The knot group $G(4_1)$ admits a presentation

$$G(4_1) = \langle x_1, x_2, x_3, x_4 \mid x_4x_2x_4^{-1}x_1^{-1},\ x_1x_2x_1^{-1}x_3^{-1},\ x_2x_4x_2^{-1}x_3^{-1}\rangle.$$

We can check easily that the following is a representation of $G(4_1)$:

$$\rho'(x_1) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix},\ \rho'(x_2) = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix},$$

$$\rho'(x_3) = \begin{pmatrix} 4 & 4 \\ 3 & 5 \end{pmatrix},\ \rho'(x_4) = \begin{pmatrix} 2 & 4 \\ 5 & 0 \end{pmatrix}.$$

Then we obtain the Alexander matrix:

$$M = \begin{pmatrix} 6 & 0 & 2t & 4t & 0 & 0 & 6t+1 & 6t \\ 0 & 6 & 5t & 0 & 0 & 0 & 0 & 6t+1 \\ 3t+1 & 3t & t & t & 6 & 0 & 0 & 0 \\ 4t & 2t+1 & 0 & t & 0 & 6 & 0 & 0 \\ 0 & 0 & 3t+1 & 3t & 6 & 0 & t & 0 \\ 0 & 0 & 4t & 2t+1 & 0 & 6 & 3t & t \end{pmatrix}$$

The numerator P of the twisted Alexander polynomial $\Delta_{G(4_1),\rho'}$ is the determinant of M_4 obtained from M by removing the last two columns. Then we get

$$P = t^4 + t^3 + 3t^2 + t + 1.$$

Moreover, we calculate the numerator of the twisted Alexander polynomials of $G(8_{21})$ for all representations $G(8_{21}) \to SL(2;\mathbb{Z}/7\mathbb{Z})$ and get 24 polynomials. These calculations are made by author's computer program and the same results are obtained by Kodama Knot program [6]. None of them can be divided by P, so we conclude that there exists no surjective homomorphisms from $G(8_{21})$ to $G(4_1)$. By similar arguments using $SL(2;\mathbb{Z}/p\mathbb{Z})$-representations for

$p = 5, 7$, we get the conclusion that there exists no surjective homomorphisms from $G(9_{12}), G(9_{24}), G(9_{39})$ to $G(4_1)$. On the other hand, 8_{18} is a periodic knot of order 2 with quotient knot 4_1. Furthermore, $G(9_{37})$ has a presentation

$$G(9_{37}) = \left\langle \begin{matrix} y_1, y_2, y_3, y_4, y_5, \\ y_6, y_7, y_8, y_9 \end{matrix} \;\middle|\; \begin{matrix} y_8y_1y_8^{-1}y_2^{-1}, y_7y_2y_7^{-1}y_3^{-1}, y_9y_4y_9^{-1}y_3^{-1}, y_3y_4y_3^{-1}y_5^{-1}, \\ y_1y_6y_1^{-1}y_5^{-1}, y_5y_6y_5^{-1}y_7^{-1}, y_2y_7y_2^{-1}y_8^{-1}, y_4y_9y_4^{-1}y_8^{-1} \end{matrix} \right\rangle$$

and the following mapping $\varphi\colon\ G(9_{37}) \to G(4_1)$ is a surjective homomorphism:

$$\varphi(y_1) = x_2, \varphi(y_2) = x_3, \varphi(y_3) = x_1x_4x_1^{-1}, \varphi(y_4) = x_3, \varphi(y_5) = x_1,$$
$$\varphi(y_6) = x_1^{-1}x_4x_1, \varphi(y_7) = x_4, \varphi(y_8) = x_1, \varphi(y_9) = x_4.$$

Similarly, we can give an explicit surjective homomorphism from the knot group $G(9_{40})$ to $G(4_1)$. Thus we have surjective homomorphisms from knot groups $G(8_{18})$, $G(9_{37})$, $G(9_{40})$ to $G(4_1)$. Hence we can determine whether or not there exists a surjective homomorphism from the group of each knot with up to 9 crossings to $G(4_1)$.

In [5], we see a complete list of whether there exists a surjective homomorphism between knot groups for 10 crossings and less.

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