

A View on Cross-Curricular Studies

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Abstract: Mathematics is an indispensable tool for other disciplines and human thought. Therefore mathematics education must reflect the importance of mathematics, and what it can do. The social and economic conditions and the education system of Turkey affect curricular studies in general. Nevertheless, introductory mathematics education is fundamental for further study in other disciplines as well as in mathematics. To equip students with mathematical reasoning and better understanding of the subject, we must let them apply, relate and discover concepts. We might as well be constructive and point out other avenues for further study. For this, we may utilize interdisciplinary subject matters in carefully designed and well-balanced courses.

Kurzreferat: *Ein Blick auf fächerübergreifende Studien.* Mathematik ist ein unentbehrliches Werkzeug für andere Disziplinen und menschliches Denken. Deshalb muß sich die Mathematikdidaktik Gedanken über die Bedeutung der Mathematik machen und darüber, was sie tun kann. In der Türkei beeinflussen soziale und ökonomische Bedingungen sowie das Bildungssystem Curriculumstudien allgemein. Nichtsdestotrotz ist eine einführende mathematische Unterweisung grundlegend für weitere Studien sowohl in anderen Disziplinen wie auch in der Mathematik. Um Studenten mit mathematischem Denken und besserem Verständnis für die Mathematik auszurüsten, müssen wir sie Begriffe anwenden, in Beziehung setzen und entdecken lassen. Wir könnten auch andere Wege für weitere Studien aufzeigen. Zu diesem Zweck eignen sich fächerübergreifende Themen in sorgfältig geplanten und gut ausgewogenen Kursen.

ZDM-Classification: B40, D10

1. Introduction

Advances in science, technology and the growth of mathematical knowledge, as well as the expectations, beliefs and habits of students and society, are forcing mathematicians to make significant changes in trends of mathematics education. The rapid development of information technologies not only has made the world a small place, but has changed the way we teach and learn.

This report is written with an emphasis on subject matters related to special aspects of the country which seem to have an impact on interdisciplinary course contents. The reader will notice similar problems are under consideration internationally. To this extent, a summary of the Turkish education system is provided and some particular studies are supplied in the first two sections. The last section is totally devoted to describing the main focus of the studies and what it should be. Yet, curricular studies are under the influence of many interrelated factors, and it is not possible to have an immediate answer.

2. Education system in Turkey

The education system in Turkey was modernized to cope with the necessities of the twentieth century shortly after the constitution of the Turkish Republic in 1923. The republic was well equipped with new teaching materials

and innovative ideas in education. Recently, the year 1997 played an important role in the history of education; the mandatory primary education of five years has been prolonged to eight years of continuous education. This contemporary event has stressed the importance of education in Turkey once again. Recent improvements in science and technology have not only forced the government to reform the education system, but also the universities to be more concerned about their educational qualities and the accomplishments of their graduates, as well as their success in their careers.

Students, after completion of at least three years of secondary school or equivalent education, can take the university entrance and university placement examinations. The nationwide examinations are compulsory for enrolling in a university. The examinations completely determine not only the university but also the particular field of study in the university. There are no mathematics or other classes that students view as a ticket to punch on their way to engineering, industrial sciences or medicine. This property of education at the university level in Turkey, necessitates a different approach to cross-curricular studies. A small fraction of students can be successful in the examinations to have the education that they wanted. Some students are unhappy with the departments they are in, and some are unwilling to learn a subject which is not of their interest. Specialization starts rather early, and that makes interdisciplinary studies more important. Multidisciplinary courses bring the chance to have more desirable careers for the future, and may even attract the interest of unwilling students to change them to be productive citizens.

2.1 Undergraduate mathematics courses

Students not only in branches of science and engineering but also in biology, life sciences, social sciences, business, economy, and in other disciplines have to take introductory mathematics courses in their first and second years. A great number of students has courses in their particular fields after their second year of education. To provide the necessary basics is the responsibility of the mathematics departments.

The course contents are mainly one and multivariable differential and integral calculus for freshmen. Sophomore level syllabi cover mainly these topics: linear algebra, solutions of ordinary differential equations, Laplace transform, Fourier analysis, difference equations, linear systems and solutions of partial differential equations. The emphasis on particular subjects differs according to specializations of students, and may have variations between universities.

Universities also offer a variety of courses in algebra, geometry, complex, functional and numerical analysis, and applied mathematics to students in different fields.

The number of years in which these students take these courses may differ to a certain extent. Moreover, most of the students majoring in engineering and physics, biological sciences, statistics, business and economics sometimes not only take their compulsory advanced mathematics courses in their third and fourth years but are also willing to take supplementary mathematics courses.

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Interdisciplinary studies cannot be avoided after the second year of university education in general. Nevertheless, undergraduate courses are designed to have a certain body of material (which may be prerequisite for a later departmental course). There are time slots and given syllabi. It will generally not be acceptable to make substantial changes for what must be seen in the first instance as an experimental teaching activity. Applications in these courses take place subject to students' restricted backgrounds in their disciplines. Still, mathematics departments need to meet the basic requirements of different departments. The emphasis on different subjects, as well as the number of class hours, may vary, and it is usually necessary to have multisectional courses.

The spectrum of courses offered to students majoring in mathematics and mathematics education differs among departments and may be dependent upon the goals of universities. It is common to have a substantial amount of courses in applied mathematics besides conventional courses in pure mathematics.

3. Curricular studies at Middle East Technical University

Middle East Technical University with around 17000 students is one of the biggest campus universities in Turkey, and offers undergraduate and graduate programs in great variety of subjects. Detailed information about Middle East Technical University, including brief descriptions of course contents, is available at the Web site at "<http://www.metu.edu.tr>"

Cross-curricular activities are found both in service and departmental courses at each level of education. Faculties are located in the vicinity of each other, and interdisciplinary studies enjoy the advantages of a campus university.

3.1 *Introductory service courses*

The mathematics department is under the big load of service courses. The department not only has to provide the basic prerequisites but also has to cope with the negative fame of having too many students fail.

The general tendency is to offer first and second year courses in coordinated large groups. Most of the time there are interactions with other departments. Usually the main concern of the other departments is the quick fulfilment of the basic mathematical methods and skills that will be useful later in particular departmental courses, and they may not always realize how much background is needed to cover these.

The first year introductory course materials can be grouped in two main categories: Calculus for basic sciences and engineering students and basic mathematics for students in architecture, city planning, biology, economics, business, business administration and international studies departments.

The basic mathematics course contents have been revised, and a new syllabus was obtained after a series of long and tiring meetings with other faculty members. The course is designed to provide prerequisites as well as applications pending students' interests. It covers linear algebra, discrete mathematics and combinatorics in addition to the

usual one and multivariable differential and integral calculus. In this course there is no time to cover the material theoretically, but it is done with considerable importance attached to conceptual understanding. The examples and exercises most of the time are chosen from real life problems. It is difficult to find one textbook to answer all the needs of this course. Therefore the substitute is to provide handouts, exercise sheets and lecture notes (see [3]).

Physics, statistics, electrical, industrial, computer and civil engineering can be named among the departments that we are mostly in contact with. In the light of the discussions there are two main groups of calculus courses, one addressing the most capable students and the other for the general audience. With an administrative decision, two subdivisions of general calculus have been made for basic sciences and engineering students. Much to our annoyance the real goal was to have us teach the same amount of calculus but with fewer credits, so that more students of engineering could spend more time taking engineering courses. It seems pertinent to note that the economics department, after these changes, made calculus compulsory instead of basic mathematics for the freshmen level.

In the second year since linear algebra, ordinary and partial differential equations are among the areas of mathematics that have been studied, applied and developed parallel to natural sciences, unification of courses for students having different majors is difficult, and discussions among different departments are inevitable. A particular trouble starts early in the allocation of systems of linear differential equations and matrix algebra, specifically eigenvalues and diagonalization, which are among the contents of two different courses offered at the same time. Usually, the asked-for order is to introduce Laplace transforms early and to carry the treatment to such a stage that the students are able to use the transform to solve initial value problems. Fourier series are expected to be covered in the first half of the course, and the net result is an ordering that is different from the mathematically natural one. It is possible to find more examples of this sort in advanced calculus and beyond the second year material.

Most of the time the perfect fulfilment of demands of the other departments depends on the availability of an enthusiastic instructor. To my mind, we can achieve better results even in a traditional program by cooperation of faculty members from different disciplines. Although syllabi are well defined, one can always find room for case study exercises.

3.2 *Undergraduate mathematics program*

One of the main objectives in an undergraduate program is to offer a well balanced education. While we want to prepare the next generation of mathematicians, there are 85 new students in the department each year, not all of them are willing to be researchers, and some have doubts about their future and job opportunities. To attract their attention towards areas of mathematics, and to equip them with mathematical reasoning, mathematical tools and abilities that they can use to their advantage for their success outside the university, the mathematics department evaluated the undergraduate curriculum. Starting from 1994, to

maximize the flexibility of the program, and the number of options for a well-rounded intellectual development of the student, the curriculum was changed to contain a large number of electives. Unlike the service courses, collaboration between faculty members with different research areas and backgrounds is easier and more fruitful. As a result, there are a number of departmentally designed elective courses with interdisciplinary contents. Students can enjoy pursuing their interests by shaping up almost half of their program with elective courses. There is also a pedagogical aspect of the program: Students attend classes not because they have to but because they want to, and can be more enthusiastic. However, to realize such a program students need formal and informal advice and detailed information about course contents.

Elective courses are designed to minimize the prerequisite requirements and have non-overlapping contents. The main catalog consists of electives that can and should be offered at all times, and there are always additional electives to introduce more modern mathematical ideas depending on the availability of instructors, student interests, and trends of the times. Among the latest ones are mathematical logic and model theory, and computer algebra (REDUCE). The innovative ideas, using technology and applications towards other areas, always find their places both in compulsory and elective courses. Students are also strongly advised to take some courses outside the department.

Today's undergraduate students do not have much background and experience in rigorous thinking. They are unaccustomed to proofs and to the strict rules of logic. The university entrance and placement examinations cover a large amount of subjects in mathematics: algebra, analytic geometry, plane geometry, combinatorics and one-variable calculus. The first year mathematics majors are good in algebraic manipulations, interpretation of the concepts, and have some conceptual intuition to start with. Partly because of the multiple choice nature of the examinations, and partly because of their school background, students are poor in graphing techniques and visualization, and they have troubles in using mathematical language to express themselves formally. Nevertheless, the examinations provide a degree of homogeneity. Students in departmental courses have a more homogenous background compared to those in service courses.

The first year serves as a foundation for further education in mathematics and has an impact on the rest of the program. It is necessary to provide a concrete introductory level background in order to ensure a sufficient quality of learning. The real world of calculus is too large, and under normal circumstances the available time is limited to cover calculus with rigor, and with an underlined importance of conceptual understanding of this powerful tool. With our majors we have the luxury of enjoying an extended calculus course for three semesters fortified with technology and applications. Owing to financial reasons, the use of sophisticated calculators is not common in classrooms, but the department has computer laboratory facilities. For the first half of the course the textbook is [4].

It is evident that industry is increasingly using aspects of

discrete mathematics, and algorithms run parallel to good theory. In addition to calculus there are discrete mathematics and analytic geometry courses to support the background. Starting from the first year, formal proofs and conceptual questions always appear in exams, homework and exercises.

Subject to the importance of teachers' roles in education, students from the education faculty majoring in mathematics education are treated equally and take most of their courses from the mathematics department. Note that education students majoring in mathematics have to have a minor subject in another area of science.

It is clear that all graduates need to have a sound knowledge of the fundamentals of mathematics, together with the ability to apply this knowledge creatively in their profession. For outstanding students of other departments there are double-major and minor programs in mathematics. The programs are designed to give sufficient background in mathematics for their future studies and research.

3.3 Evaluation of the program

The realization of curricular changes is influenced by many complex factors. To have fundamental changes in the structure of the undergraduate program calls for a close evaluation. Questionnaires addressed to both faculty and students are the main tools. In the presence of new ideas and conflicts it is common to have departmental meetings. It is among the routines of the department to have meetings with students at each level and to listen their points of view. Graduates are also welcome to share their opinions with faculty members.

As in every reformist approach, there are still unsettled points, and certain issues are under discussion. Some problems are compounded by the unusual length of first-year calculus and the uneven distribution of subjects in favor of algebra and analysis. It is only by a recent change that differential geometry has become a compulsory course, and that a partial differential equations course is under discussion. There can also be doubts about the right of existence of some courses in the program due to their similarities of content with courses offered by other departments.

Another aspect of the studies is the difficulty of choosing a suitable textbook. Apart from finding a sensible book that discusses all the subject matter there are financial limitations on the students' side. However, there is always a main textbook for every undergraduate course, and auxiliary materials are available to students. It is a common belief that well designed syllabi and clear goals for courses are necessary, and textbooks are crucial in the realization of such goals.

Nevertheless, there are some satisfactory results. Sophomores and juniors have become more enthusiastic and competent. Our graduates are considerably successful in finding jobs in surprisingly diverse areas. They are quick in learning and adapting to new situations, and they are well aware that competence in mathematics is the main reason for their success.

Educational studies at the undergraduate level are done under the coordination of a committee and by "department traditions". The lack of an undergraduate director makes

the level of studies rather amateurish and not exactly professional. There is always the risk of falling into the same mistakes, and passing through the same patterns.

4. Interdisciplinary course contents

Mathematics is a powerful tool, it is indispensable for mankind to think and reason, and in practice all branches of mathematics at all levels serve as a tool for all parts of science and for many disciplines other than science and mathematics. We must point to the great advantage for students in their own lives to be able to reason effectively and solve quantitative problems, and we might as well be constructive and provide avenues for further studies.

Thurston (see [7]) points out that mathematics is a tall and a broad subject. It is tall because concepts build on previous concepts. It is broad in the sense that it is a highly diverse and interactive mixture. On the other hand, while mathematics develops deductively by an axiomatic approach, the human brain learns more inductively. Students might well feel more comfortable to begin with coherent examples and work from them to general principles, from simple to complex. The traditional subject oriented teaching no longer seems to be suitable for the satisfactory development of students, and to answer the needs of the society. In our times the curricula contents have changed in a healthy manner towards the goal of conceptual understanding, rather than providing students with well-polished proofs and established algorithms leading to memorizing instead of learning. Creativity of the mind is trained through discovering new ideas.

Realistic applications to other disciplines are among the tools to provide intuition and to improve general reasoning abilities. Problem solving abilities might be improved by mathematical modelling as well, and might be more relevant to students. The essence of using applications in the classroom and interdisciplinary knowledge should be more than mere mastery of facts and routine skills: It should require students to understand and apply mathematical concepts in new situations.

To have a balanced conception of mathematics, including the aspects of theory, application and mathematical modelling may cause the curricula to be rebuilt completely. To fit interdisciplinary course contents in a sensible syllabus, and use applications and modelling for their full affects calls for a tedious and careful work. Before making radical changes in curricula we must define our goals clearly, diagnose the problems, and determine feasible solutions. Training of teachers and cooperation with other disciplines might be necessary as well. Yet, we do not know how, and we have not realized all potentials. Parallel to the advances in science and technology curricula studies will continue to have an open end.

4.1 Introductory topics and applications

After the development of calculus by Leibniz and Newton, much of modern mathematics is derived from calculus, and many of the ideas in differential geometry, statistics, and numerical analysis are descended from the study of calculus. Due to advances in science we may see that several mathematical ideas are located in non mathematical disciplines. It is well known that physics and technol-

ogy are at the origin of certain theories, such as harmonic analysis, spectral theory, partial differential equations, and information theory. In other cases physics has adopted and popularized mathematical theories where these have proved useful long after their first occurrence. Today chemistry uses algebraic topology, and coding techniques use Galois fields. Technological advances also helped many areas as well, such as mathematics of dynamical systems, and the ideas, representations, and skills needed to make sense of nonlinear phenomena to grow in a healthy manner (see [2] and [6]).

Constant changes within and between different disciplines result in a rather inconsistent picture. However, the mathematics curriculum cannot afford to be over-full and lacking balance and coherence. The teaching of introductory mathematics must focus on concepts that are easy to understand and could lead to a better understanding of the nature and power of mathematics and human thinking. Mathematicians cannot afford to be inaccurate, but we can simplify, and for better understanding of its power and relevance with the real life we may use illustrations from other subject matters.

While arithmetic deals with sums, differences, products, and quotients, calculus deals with derivatives and integrals. Derivative and integral theory have a variety of applications to physical, life and social sciences. In our times a lot of applications from mechanics in physics are among the contents of course materials, and are covered commonly in standard calculus texts. Still there are many subjects that are never discussed in the mainstream curriculum.

A balance can be obtained in conceptual understanding and manipulative skills by using real life problems. There are key underlying ideas: constant rate, mean value, and area under a graph. Understanding derivatives as rates is a standard tenet of calculus; ideas of rate of change, accumulation, the connection between variable rates and accumulation, and accumulation must be made intelligible to students.

The area under a graph, and examples from commonplace financial transactions (eg. present value) with a careful evolution from the discrete case to the continuous, may provide a conceptual insight into the definition of the definite integral as the limit of Riemann sums. On the other hand, subjects of vector calculus arise in quite different ways in physics and economics. When calculus was first developed it was based on an intuitive notion of infinitesimals. Recently, infinitesimals have had exciting applications outside mathematics, notably in the field of economics, and it is quite natural to use infinitesimals in modelling physical and social processes. Technology also allows us to incorporate linear regression, Markov processes, and probability distributions in an introductory calculus course.

The problem is to determine how aspects of disciplines may be integrated to form a coherent course whilst at the same time restricting its contents to a realistic level. There is considerable enthusiasm for infusing the first and second year curriculum with more applied material. Among the first refreshing examples of the new approach are the

Harvard (see [5]) and Amherst project (see [1]) materials.

Trying to understand the world around him, man organizes his observations and ideas into conceptual frames. Most of the time the purpose of analysis of a physical, chemical, biological, economic, or social model is a prediction of the behaviour of a system. Often a system has yet to be built, but is nevertheless a system which will eventually exist in the real world. Between the real-world system and the solution there is an abstraction called a mathematical model. If all that is required is a general picture of the response of a system to small disturbances, then a linear model may well be sufficient. An exact method may be available to solve such a model. However, if an accurate prediction of the behaviour of a system under extreme conditions is required, then the model must be a closer representation of reality. It is always necessary to distinguish very clearly between the model and the part of the outside world that it is supposed to represent. The best model is the one which yields a solution describing the most realistic picture of the real event under consideration.

On the other hand, the problem is said to be properly posed if it satisfies the following requirements, under given conditions. *Existence*: there is at least one solution. *Uniqueness*: there is at most one solution. *Continuity*: the solution depends continuously on the data. The first requirement is an obvious logical condition, but we must keep in mind that we cannot simply state that the mathematical problem has a solution just because the real problem has a solution. We may well be erroneously developing a mathematical model consisting of an ordinary differential equation whose solution may not exist at all. The same can be said about the uniqueness requirement.

The pair “model and reality” play an important part in human thought, and the underlying notions and mental operations can best be learned by building relevant models and applying theory to their use. A good number of applications can be located within the theory of ordinary and partial differential equations and linear algebra. The nature of the subjects underlines the principles of rational approach in solving problems. Mathematics also provides tools for analyzing models analytically, graphically and by numerical methods. With the advances in technology, it is possible to use real data and have more realistic applications. Nevertheless, well known mathematical models of population dynamics, Malthusian law, logistic equation, Lotka-Volterra predator prey model, etc. and celestial mechanics are among the classical examples. It is possible to encounter them early in a calculus course.

Linear algebra is central in pure and applied mathematics and as applicable as calculus. It is an essential part of the toolkit required in the modern study of many areas in the behavioral, natural, physical, and social sciences, in engineering, in business, in computer science, and of course in pure and applied mathematics. Linear algebra allows and even encourages a very satisfactory combination of elements of mathematics, abstraction and applications. It is possible to find classical examples of this sort in other subjects. One of them is the theory of complex functions. Decades ago, this was the central point of mathematics

teaching, and it still has important applications today.

Linking basic concepts of mathematics to ideas or activities in another discipline will surely improve students' ability to learn and understand a subject in an integrated way. We should let the students apply, relate and discover, and applications are tools for this, but they are not the inner nature or the most important quality of mathematics.

4.2 A look into

The interdisciplinary subject contents should not result in “math avoidance”. We must be careful about what we want to convey. We have to consider that mathematical modeling is complicated and difficult. Applications should not be full of irrelevant details of some other subject.

We must avoid to simplify examples to an unrealistic stage. To describe derivative and integral in terms of an automobile trip, and to explain that the speedometer reading is the rate of change of the distance is boring. Fencing animals in rectangular pens or to say a student can get a score of $\frac{100t}{t+1}$ on his mathematics exam if he studies t hours, and if he studies infinitely long his score will be infinitely close to 100 are unmotivating examples. It is even worse to define e as “bank balance after one year of one dollar invested at an interest rate of 100% compounded continuously”.

After all we don't live in a two-dimensional world, and a derivative as the best linear approximation to the function near a point is only one of the ways. The exponential function is the inverse of $\ln x$, but the most important fact about the exponential function is that its derivative is equal to itself. Lebesgue's definition of integral is intuitively simple, and can be stated without a lot of machinery. The choice of applications must include cognitive complexity and should lead to dealing with topics that students will see as interesting, relevant, or exciting, and must be worked out with care and lead to deeper understanding and appreciation of the concepts.

We must keep in mind to challenge students' thinking abilities to a realistic end. Experts believe that achievement is based on cognitive maturation. Freshmen or sophomores need time for full understanding of new or intertwined ideas. To this end, carefully chosen applications leading to discovery of concepts can best be used in homework and case study exercises. Group homework can be helpful for better understanding, as students may see ideas from several different approaches, share ideas and learn cooperatively. We can find more room for less conventional and open ended problems in homework.

Textbooks are another aspect of curricular studies. They are essential, and they should reflect the main focus of the subject and be well organized. Unfortunately, for the sake of infusing interdisciplinary contents, some textbooks have grown to enormous number of pages, have obscured the spirit of mathematics, and some of them have misused mathematical language. Students give right reactions to right attitudes.

Last but not least, introductory courses and curricula must reflect the necessity and importance of having a strong background for further studies in mathematics or outside it. Subjects that serve as a starting point not only

must have their place in the programs but also we cannot leave fundamental ideas without their having been understood. As it was mentioned in section 3.3, the emphasis on differential geometry and topology has decreased in undergraduate programs, and because of the multiple choice nature of University entrance and placement examinations, the study of solid, spherical and plane geometry has lost its real importance in school education. So we leave most of the students blind without geometric intuition and visualization.

4.3 Conclusion

For decades mathematicians have written and talked about the importance of mathematics for human thought, but the viewpoints and demands of society show that they cannot have been convincing. We must keep in mind that the dismay of society towards mathematics can only be cured by training generations and teachers who have a large base of knowledge in mathematics, and who understand and enjoy its power. To connect mathematics with real life and with people must be the starting point of the studies.

Changing the old habits in teaching and public attitudes towards mathematics may require a long and concerted effort by many committed people. For a teaching faculty member, doing research and having other responsibilities is exhausting. To deal with educational studies is important and has more merit than has been acknowledged. Better results can be obtained through good motivation. Universities must attach as much importance to educational studies as they have attached to research.

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