Upitis, R.; Phillips, E.; Higginson, W.:

Creative Mathematics
Exploring Children's Understanding

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Creative Mathematics is a delightful story of collaboration, mathematical curiosity, and liberal learning. The collaboration reported here is of the toughest sort – a classroom teacher, Eileen Phillips, and a university researcher, Rena Upitis, Dean of the Faculty of Education at Queen's University in Kingston, Canada; it is enriched by the further collaboration of Bill Higginson, Co-ordinator of the Mathematics, Science, and Technology Group at Queen's. Higginson, acting at a distance, contributed advice to the pair working in the classroom and commentary on their written account. Phillips and Upitis worked with third and fourth grade children in Vancouver while Upitis was on leave from Queen's; Higginson encouraged, advised, and commented.

Mathematical curiosity infected both the children and their teachers. It is clear from the start that Phillips is a fine teacher, but she had never before tackled the mathematics reported here. Upitis is a music specialist and was lured into the project in part by promises of connections between math and music. Higginson is something of a polymath mathematician, one interested in math and its connections to almost everything. The children, none of whom had listed math as their favorite subject at the beginning of the school year, became eager for math time and sorry when it ended. Everyone involved seems to have become more, not less, enthusiastic as the year progressed.

The math learning was liberal in the best sense. The activities offered set the children free to investigate matters that excited them, and they learned a good deal more than the usual arithmetic. Their projects took them into art, music, technology, and invention. Along the way, their vocabularies grew impressively.

Each author provides an introduction and commentary in each chapter. Upitis' many conversations with Higginson led her to explore the connection between music and mathematics. Confessing that she had been a proficient but not particularly interested math student, Upitis reports being converted by the fascinating titles in Higginson's mathematics library. (I have seen that library and understand Upitis' surprise and delight.) Upitis notes:

"It is not as if these books do not talk about algebra and geometry – they do. It is that these books do not use algebra and geometry as the endpoints in mathematics, but rather, as tools for creation – tools that are needed to make beautiful tessellating patterns or to understand the allure of a snowflake" (p. 2).

With Higginson's inspiration and the cooperation of university colleagues in Vancouver, Upitis set out to find a teacher – collaborator.

Phillips, in her introduction, expresses fears common to classroom teachers – concern about tackling unknown subject matter, uneasiness about working with a university researcher, and worry over an already busy teaching life. Besides, she was a successful teacher by all the usual standards. Why, then, should she complicate her professional life by plunging into an alien project? However, she was inspired and reassured by Upitis and, further, she reports that something had been bothering her for a while:

> "I felt that I was holding back. I was focusing too much on being accountable to people outside the classroom. I wanted to focus on the pupils and on what they might need to know. Even when I moved to a combination of textbook work with manipulative materials and methods from my early days, I remained unsatisfied." (p. 8)

These remarks struck me as especially important for American (U.S.) teachers. With the current emphasis on higher achievement scores, many U.S. teachers feel that accountability is pointed in the wrong direction. Instead of being accountable to their students, teachers are forced to consider the demands of administrators and a public only partly aware of the ramifications of their demands. As I travel around my country these days, I hear teachers everywhere complaining that they have been forced to teach in a way that kills interest and does little to further understanding. At the same time, of course, they are urged to teach for understanding.

Phillips exudes a wisdom rarely seen in classroom teaching. Watching the pleasure and enthusiasm of her students, she becomes dedicated to new approaches and topics. But in response to the concerns of parents ("When will you teach real math?"), she wisely combines textbook exercises, work sheets, and projects. She knows how to accommodate a host of conflicting needs and interests. I found her attitude refreshing. After all, we really do not know how much practice is required for children to learn the skills they need to address significant mathematical problems. Alan Schoenfeld remarked on this issue that we do not know "the degree of fluency required to do competent work" (1994, p. 60). Thus, Phillips was not simply accommodating concerns that might have had a political impact on her work as a teacher, but she was also sensitive to the actual needs of her students. Neither Phillips nor Schoenfeld subscribe to the debilitating notion that students need to learn a mass of rote skills before tackling hard and interesting problems, but both recognize that many skills have to be acquired somewhere, sometime, some way, if students are to be successful in using mathematics. Indeed, when the children were working with a project on animation, one child commented, "It's a good thing we know our eight times tables" (p. 65).

Higginson, in recounting his student days in mathematics, tells a familiar story. He pleased his teachers and out-performed his peers. How many of us could tell this story! (And how many simply gave up because they could not "out-perform" their peers?) It was not until graduate school that he became thoroughly engaged in mathematics. Now, of course, it is a matter of professional concern for him to find ways to make mathematics accessible and interesting to a great number of students.

Thus, three thoroughly motivated people launched an impressive project. In the following chapters, they describe how children worked on tessellations, animation, mathematical "jewels" made from paper, kaleidoscopes, and musical composition. They learned to look for patterns in the world (seeing tessellations everywhere), came to a working definition of tessellation ("no floor showing"), made important generalizations (e.g., "all quadrilaterals will tessellate"), used symbols to describe their patterns, applied the basic skills they had acquired earlier, and extended their vocabularies. Picture little kids easily using words such as tessellation, animation, trapezoid, hexagon, quadrilateral, and thaumatrope.

Phillips and Upitis put considerable emphasis on mathematical communication. This is a familiar focus today, but their approach is more sophisticated than most. The children were encouraged to describe their work in symbols so that others could re-create the patterns they had constructed, and they largely succeeded. I found this work impressive, and it contrasts sharply with many examples of mathematical communication found in various portfolios. The difference, and it is an important one, may be in the time spent on these projects. The children were not rushed, and the communication was undertaken as important in itself; it was not pro forma and, because they were asked to invent symbolic forms of communication, it added considerably to their mathematical knowledge.

Typically, after Phillips and Upitis describe what happened in the classroom, Higginson offers a commentary loaded with helpful references. There is clearly a lot of help available for teachers who want to engage in the kind of activities described here. I was reminded, as I read these accounts, of the optimism and good sense that accompanied the best of Open Education. Other readers, too, might want to revisit that literature. (For a personal/historical narrative on Open Education, see F. Hawkins, 1997.)

Interestingly, although the authors do list work by David Hawkins in their references, they do not mention Open Education. Instead, and I think this is a strategic error, they draw an analogy between their approach to mathematics and the whole-language approach to reading and language arts. This is a strategic error because whole-language has come under devastating fire in recent years (at least in the U.S.), and anyone who wishes to use it as an example needs to defend it. I believe that a persuasive defense can be offered, but the best use of whole-language would probably be laid out in terms of accommodation and eclecticism – as Phillips described her own approach earlier. Indeed, a bit of critical educational history would have been a welcome addition to the book.

The question of what constitutes "real" problems for students arises several times, and it is sensitively discussed. In the preface, David Pimm agrees with my comment on "real-world" problems – that "a problem that is 'real-world' in the sense that adult human beings grapple with it may not be 'real' at all in the school setting" (Noddings, 1994; Pimm, xii). Both Pimm and the authors clearly locate the meaning of "real" in what is real for children – activities that engage them and from which they learn. Thus, the "real" presents a challenge worth meeting. It can be fun, but it is not merely fun. Working on real problems induces growth. On this, the authors are demonstrably Deweyan, although they do not refer to Dewey.

Higginson's comments on technology are eminently worth reading. They are even-handed and sensitive. He notes the promise of technology but also its downside. Students may bog down in trivia, pursuing this and that bit of information without studying anything in depth. They may feel compelled to use technology even when their own styles suggest a different approach. On this, Higginson's report of an interview with the computer scientist Donald Knuth is heartening. Knuth finds e-mail counterproductive for his work. He acknowledges the usefulness of electronic mail for many occupations, but not for this own! Comments like these can be enormously liberating. On the positive side, Higginson's account of what can be found on the Web may send people scurrying for useful information. Consonant with his even-handed analysis and his love of books, Higginson finishes by saying, "As we explore the Internet in the years to come, we would do well to remind ourselves to search the shelves of second-hand bookshops as well" (p. 130).

The last chapter, "Children as Mathematicians," raises several important issues. The authors frankly discuss a few things that didn't work. Among the unsuccessful projects was one on codes. I was surprised by this because I recall codes as one of the most successful topics I used with sixth graders many years ago. Maybe the children need to be a bit older. Maybe the teachers did not uncover the underlying interest that might have motivated such a unit of study. Maybe they were not all that interested themselves. I remember that my students were interested in secret languages, writing with disappearing ink, and that sort of thing. From there, we began to do cryptograms and then went on to read a Sherlock Holmes story, "The Adventure of the Dancing Men" and Poe's "Gold Bug", both of which involve codes. I remember the unit as great fun, and the children learned quite a lot about the frequencies with which various letters appear in English and about letter combinations.

Children differ in their interests; so do teachers. That is an important message in the last chapter. There is no plausible reason why every group should be interested in codes or tessellations or animations. However, a teacher's enthusiasm may infect the children. At least, that enthusiasm will tend to support the teacher through the hard work of introducing and successfully completing the kind of rich projects described here.

My own mathematical interests tend to lie in the logical and literary. I wondered, as I read this fascinating report, whether I would enjoy doing tessellations with children. Maybe. But I know that I enjoy sharing and sorting out the logical puzzles and anomalies in "Alice in Wonderland" with students of all ages. And I am always looking for stories that include mathematics in any form. Not long ago I discovered the short stories of the Japanese writer Kenji Miyazawa and found that numbers appear in almost every one. My favorite is the story of General Son Ba-yu. A physician examining the General learns that his patient has been bewitched "about ten times." The diagnostic session proceeds from that disclosure:

"I'd like to ask you something, then. What does one hundred and one hundred make?"
"One hundred and eighty."
"And two hundred and two hundred?"
"Let's see. Three hundred and sixty, if I'm not mistaken"
"Just one more, then. What's ten time two?"
"Eighteen, of course." (Miyazawa 1993, p. 25–26)

At this point, the doctor knows just what to do and, after treating the General, repeats the original questions. Now the General answers correctly, and the physician says, "You're cured. Something was blocked up in your head, and you were ten percent off in everything" (p. 20). Perhaps it is stories such as these that make Japanese students so good at mathematics!

My point in introducing my own interests in logic and literature is to remind readers that our interests differ. Teachers and researchers who are not enthusiastic about one set of topics can surely find another, and successful completion of one project will almost certainly lead to further exploration. Teachers, like children, may build on their current interests and move well beyond them as their confidence grows.

I may differ with the authors on one important issue. I'm not sure that students have to learn "to think like mathematicians." I concede that it is the popular view today in informed circles. But why? If we were to take the advice of all the disciplinary specialists, our children would have to think like scientists, artists, historians, writers, linguists, musicians, geographers, literary critics, dramatists, editors, and mathematicians. I'm not sure this is reasonable. It does seem reasonable, however, to analyze these ways of thinking for those elements that students might profitably adapt to their own purposes. I agree whole-heartedly that students should be able *to use* the mathematics they learn but surely the uses will vary with interests and talents. We may be deluding ourselves when we try to capture the complexity of learning to use mathematics by reducing it to "thinking like a mathematician." This book provides strong evidence that kids can use mathematics while doing work similar to that of artists, engineers, film makers, or composers. In none of these cases, however, do the children take on the complex modes of thinking, semi-permanent attitudes, and social networks characteristic of professionals in the field. For kids, doing real mathematics involves using mathematics on problems of interest to kids. Fortunately, we can almost always find such problems, and their mastery may well lead to new levels of reality. Upitis, Phillips, and Higginson have demonstrated what can be done.

References

Hawkins, F.: Journey with children: The autobiography of a teacher. – Niwot, Colorado: University Press of Colorado, 1997
Miyazawa, K.: Once and forever. – Tokyo: Kodansha International, 1993
Noddings, N.: Does everybody count? – In: The Journal of Mathematical Behavior 13(1994), p. 89–104
Schoenfeld, A.: What do we know about curriculum? – In: The Journal of Mathematical Behavior 13(1994), p. 55–80

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