FROM DIVERSITY TO INCLUSION AND BACK: LENSES ON LEARNING *

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The ideas presented in this lecture are based on the observation of processes of construction and consolidation of knowledge by individual students learning in groups within classrooms along a sequence of activities. Whereas the uniformity of the basic elements used to describe the knowledge construction processes may be seen as inclusive, there is a lot of diversity in the different ways in which individual students combine these basic elements into their personal learning trajectories.

INTRODUCTION

There is a thread, which links this plenary talk to the one I gave at PME 23 (Hershkowitz, 1999). In the previous one I asked, "Where in shared knowledge, is the individual knowledge hidden?". My point was that research should focus more extensively on the investigation of the development of individuals when they evolve in different social settings and construct of knowledge about different topics through successive activities.

At that time many researchers in mathematics education were attracted by the investigation of the construction of the "shared knowledge" of a community of students (e.g. Cobb, 1998; Hershkowitz, and Schwarz, 1999). Most researchers' lenses were focused on the ensemble. Individuals were observed as "members" and the knowledge of the individual was seen as a contribution that transformed the knowledge of the ensemble, where the ensemble designates "the smallest group of individuals who directly interact with one another during developmental processes related to a specific activity context" (Granot, 1998). Research on shared knowledge was mostly based on the interpretation of various episodes in different social settings. The episodes were mostly taken from one lesson, and even when the sample of episodes where taken from a sequence of activities the data were accumulated by observing different ensembles within the classroom, populated by different students with no possibility to trace the learning trajectory of specific students along a sequence of activities.

Less research effort has been invested in the opposite direction, namely in investigating the shared knowledge, constructed by a group of students or by a dyad, with the aim to better understand the development of the participating students' individual knowledge. The work of Kieran and Dreyfus (1998) is an example in this opposite direction. Kieran and Dreyfus observed student dyads solving problems, and

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then right away interviewed each student individually using an analogous problem in order to check the effect of the dyad work on each individual.

Recently I have invested efforts with colleagues and students in this opposite direction, to describe and understand how individuals construct new (to them) structures of knowledge in peer interaction and consolidate it (or not) in subsequent activities. We did not observe students or small groups when isolated in laboratory conditions; rather, we traced the participation of individuals and groups by studying talk in the classroom. This was done along a sequence of activities (Tabach, 2001; Shtein, 2003).

Our empirical approach led us to focus primarily on process aspects of construction of a new structure of knowledge rather than on outcomes. We focused on a particular kind of construction of knowledge, the process of abstraction, which we defined as a process in which students vertically reorganize previously constructed mathematics into a new mathematical structure. In order to empirically study abstraction, we looked for observable actions relevant to the construction of knowledge. Following Pontecorvo and Girardet (1993), we called these actions epistemic actions. We identified three epistemic actions relevant for processes of abstraction: Recognizing, Building-With, and Constructing, or short RBC. Two case studies in which we observed students evolving in laboratory settings led us to initiate the elaboration of a model of abstraction: we started with an interview with a single student (Hershkowitz, Schwarz & Dreyfus, 2001), and then turned to the observation of dyads working in collaboration. In the second case study, the shared knowledge of the dyad and the construction of a new structure of knowledge of each individual in the dyad were investigated by analyzing pair interactions between the two students. Interaction was investigated in detail as a main contextual factor determining the process of abstraction (Dreyfus, Hershkowitz & Schwarz, 2001a). A crucial feature of the model is that the epistemic actions are nested within each other. We therefore called it the nested epistemic actions model of abstraction in context, but usually refer to it simply as the "RBC-model". The model is described in detail in these references. Shorter descriptions may be found in PME proceedings (e.g., Dreyfus, Hershkowitz & Schwarz, 2001b).

We were aware that the contexts in which the model was elaborated were quite limited. Social interactions and other contextual factors in school classrooms are often much more complex than in research interview situations. Therefore, we began about two years ago to expand our program of research in two directions. The first one is concerned with the construction of knowledge in teacher-led whole-class discussions. We initially focused on the role of the teacher (Schwarz, Dreyfus, Hadas & Hershkowitz, 2004). The second direction is at the heart of learning and development: We decided to develop theoretical and experimental tools to follow individuals participating in successive school activities such as collaborative problem solving sessions or individual problem reporting, in order to possibly identify construction or abstraction of the individual in a wider time-scale. One of the main questions we asked was whether it is possible to speak about consolidation (or its opposite: fragmentation) of knowledge along a sequence of activities.

DIVERSITY AND INCLUSION: A SOCIO-CULTURAL PERSPECTIVE ON THE PSYCHOLOGY OF MATHEMATICS EDUCATION

The dialectical approach I adopted (with many other math educators in our community) is exemplified in this plenary: we investigate how shared knowledge is constructed and, to do this, we need to go back to research on knowledge construction by the individual. However, this individual is not isolated like in a laboratory; she or he learns in a context, and the researcher constantly faces the "problématique" of isolating and investigating the development of individual knowledge within the shared knowledge of a changing/developing community.

The link of my personal interest to the interest of my changing community, the PME community in this conference, is then, I think, quite obvious. I would like in this plenary session to focus on *diversity and inclusion* of learning processes within a group of individuals, and to express it via the RBC model of abstraction. I will present data from two girls who participate (actively or passively) in the same class dialogues, and collaborate in the same small group (a group of three). The different combinations of constructions of knowledge, whose trajectories vary from one girl to the other, show *diversity* within a group of individuals. On the other hand the expression of this diversity and its analysis for each girl are based on the use in the same three epistemic actions, as they are reciprocally nested among them. We may relate to these basic ingredients, which characterize abstraction processes, as to the *inclusion* of these processes. Individuals will have also different ways of consolidating what they abstracted earlier -- we are again facing *diversity*.

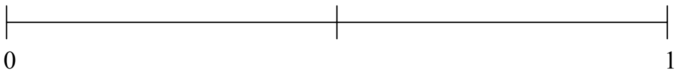
Let me provide an analogy to clarify this idea of *diversity and inclusion* in learning theories. Let's think about the relevance of a "good" micro-world to learning. It provides well-defined primitives that are easy to use. These primitives afford "*inclusion*" because they are the same and provide the same learning opportunities for each learner. However, this *inclusion* has the potential to produce, within a community of learners, a *diversity* of ways to solve a given problem and this diversity is due to the *inclusion* the tool affords. Because the primitives are easy to manipulate, the learners can use them to produce many different combinations, each of which expresses a different way to solve a given problem. Like any analogy, this one has its limits: while the primitives of the technological tools are designed beforehand by designers and are made visible, the observability of RBC actions as primitives of abstraction depends on our judgment.

In the following sections I will use data from the two girls mentioned above to reflect on inclusion and diversity in the above sense by analyzing the intertwining of RBC combinations nested in each other along a sequence of tasks.

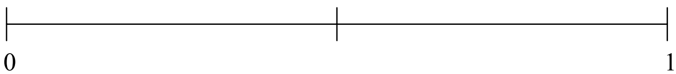
THE RESEARCH SETTING

The research took place in grade 8 classrooms during an 8-lesson unit on probability, organized in five activities including tasks for small group collaborative work and for whole-class discussions. The activities were designed so as to create opportunities for construction of knowledge. One set of tasks in the second activity was designed to introduce students to issues related to repeated events, by asking them to locate the probability of various repeated events on a chance bar (tasks 5, 6, 7 and 11). Students come back to this issue in a written (individual) final quiz of the unit (task Q2) as well as in an individual interview (task T3). These tasks are presented next.

5. You spin a Chanuka dreidel 100 times (the letters that appear are **N**, **G**, **H**, **P**). Mark approximately, on the chance bar, the letter that designates each event, and explain:
   A: The outcome will be **N** all 100 times.
   B: The outcome will never be **N**
   C: The outcome will be **N** between 80 and 90 times.
   D: The outcome will be **N** between 20 and 30 times.
   E: The outcome will be **N** exactly 25 times.
   F: The outcome will be **N** exactly 26 times.

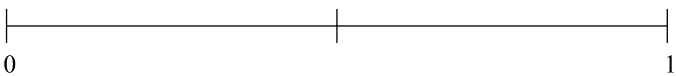


6. You flip a coin 1000 times. Mark, approximately, on the chance bar, the letter that designates each event, and explain:
   A: The outcome will be heads all 1000 times.
   B: The outcome will be heads between 450 and 550 times.
   C: The outcome will be heads between 850 and 950 times.
   D: The outcome will never be heads.



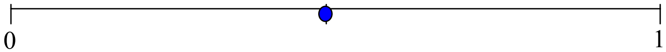
7. Which of the following events has a bigger chance to occur? Mark, approximately, on the chance bar, and explain:

   A: The outcome will be heads between 450 and 550 times.
   B: The outcome will be heads exactly 500 times.



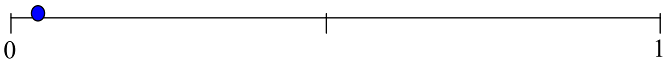
11. A regular die was thrown 100 times and the students were asked to mark, approximately, on the chance bar the probability to obtain 50 times an even number.

Amir marked the middle of the chance bar:



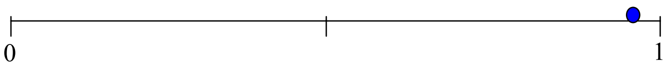
Amir explained: *I marked it this way because the chance to get an even number is **one half**.*

Shira marked a position close to zero on the chance bar:



Shira explained: *The chance to get an even number is one half; but **no way** will there be **exactly** 50 times even. Maybe there will be only 46 times even, or 52.*

Nir marked a position close to one on the chance bar.

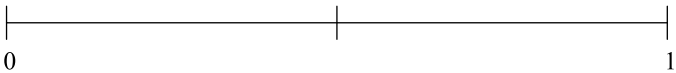


Nir explained: *I marked close to 1 because the chance to get an even number is one half. Therefore in half of the throws, the outcome will be even and thus it is **almost certain** that there will be 50 even numbers.*

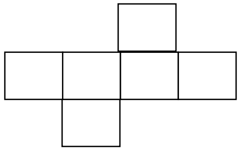
Who, do you think, is right? Explain!

Q2. You throw a die 1200 times. Mark approximately, on the chance bar, the letter that designates each event:

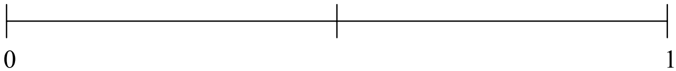
A: The outcome will be 6 exactly 200 times.
B: The outcome will be 6 exactly 202 times.
C: The outcome will be 6 between 100 and 300 times.



T3. In the eighth grade booth at the school fair, they use various dice for chance games. The flat view of one of the dice is. This die is thrown 900 times. Mark, approximately, on the chance bar the letters that designate events A and B:



A: The outcome will be 1 exactly 450 times.
B: The outcome will be 1 between 400 and 500 times.



Answering these questions requires a number of knowledge structures; however, in order to keep the discussion focused, I will focus on the construction of the following principles: *The probability that the frequency of an outcome is in a specific range of given length is large if the range includes the expected value, and small if the range is far from the expected value. The probability that the frequency of an outcome is equal to the expected value itself is very low.* Most items listed above require these principles at least partially. The remaining ones (5A, 5B, 5F, 6A, and Q2B) have been added for completeness and coherence. For brevity, I will refer to these principles as “the focus knowledge structure”.

At the beginning of tasks 5 and 6, students are given opportunities to learn that *the probability of the frequency of a repeated event is smaller than the probability of the corresponding simple event, and that the probability of the frequency of a repeated event decreases as the number of repetitions increases*. Schwarz et al. (2004) analyze the role of the teacher in a detailed discussion on items 5A and 5B. They also show how the difference between the probability of a simple event and the probability of the corresponding repeated event is constructed as shared knowledge about relative frequency in the classroom. In the sequel, I will refer to this as “the preliminary knowledge structure”. This construction of the preliminary knowledge structure is an epistemic action in its own right, and we will see that it is nested in the construction of the focus knowledge structure.

In the following subsections, I will present a classroom discussion of tasks 5C, 5D and 5E. Then I shall focus on two girls, Yael and Rachel as they participate in subsequent activities.

5C: The outcome will be N **between 80 and 90 times**

Guy marks C at 1/4 on the chance bar.

Yael 83: *It’s much less than he marked. It’s close to B. It can’t be a chance of 1/4 that it happens…, it’s not…*

Ayelet 84: *That N comes up between 80 and 90 times means that the other three letters come up between 10 and 20 times; that’s much less than what Guy marked.*

Yael 85: *It's much closer to B. A little larger than the B but very close to it, like the distance between A and B.*

We can see here that the students who participate in this discussion agree that the probability that the frequency of an outcome in a given range, that is far from the expected value, is small. The shared construction of this part of the focus knowledge structure appears to be unproblematic for the students. We presume that this is so because its low probability conforms to the low probability in the preliminary knowledge structure.

5D: The outcome will be N **between 20 and 30 times**

Eliana goes to the board and marks a point close to 1/4

Adi 93: *I think … 30%*

Adi 95: *There is a greater chance …*

Adi 97: *It's closer to the middle.*

Teacher98: *Does somebody have a different impression, wants to support or object? What do you think, Guy?*

Guy 99: *I think it is much higher.* [Teacher asks how much.] *80%, because there are 4 sides, right? And the chance it falls on one of them is 25%, and you said it falls between 20 and 30, so …*

Yael 100: *Thus it is 25%. It's not 80%.*

Guy 101: *No, that it falls on this 25 times, on this … out of 100 … 80, about 90%.*

Guy 103: *Just a second, can I continue this? It's not how many times the outcome …*

Omri 104: *What I'm trying to see, if I understood Guy: that there is one chance in four … thus that there is a very high percentage that it will be between 20 and 30.*

Omri 108: *What he says is that every time you spin, there is a chance of one in four that it will fall on N. In other words, now 25% out of 100 that's about the number of times it will fall on N. That's a very high chance.*

Teacher 109: [To the class:] *What do you think?* [To Rachel:] *You nod your head – with whom do you agree?*

Rachel 110: *With Guy.*

Michael 111: *Guy is right. As Omri says, it's not sure that if you spin once, it will come out 1/4. More times you spin, there is a greater chance.*

Itamar 112: *I agree with Guy, it is 75%. If I say that's 25 it's once, then maybe because I think it's high I deduct 25% from the certain.*

Yael 115: *I am still not sure. Guy succeeded in convincing me, but in the beginning I thought it was half but still …*

Teacher 122: *That is you expect an answer between 20 and 30; that's something we expect will happen. Thus, if that's what we expect to happen, then the chance is large, close to 1.*

In 5D, the students face two challenges: The first concerns the fact that for the first time they see a case in which the probability of a repeated event is close to 1. The second challenge is that the range includes the expected value. The second challenge naturally invites students to mark the probability of the simple event at 1/4. The class as a community seems to construct a new structure of knowledge, another part of the focus knowledge structure. That such a construction has indeed occurred can be inferred, for example, if the structure is being used in later tasks. This is exactly what I will show. I will focus now on the two girls Yael and Rachel, who always collaborated when the class was asked to work in small groups. I first reflect on their participation in the class discussion. I will then trace their behavior in subsequent activities.

Yael

We first follow Yael in the class discussion. Yael marks the probability for events 5A and 5B very close to 0 on the chance bar. She is not very active during the discussion on these questions. However, later on, during group work on task 6, she uses explanations raised during the class discussion, in order to convince Rachel. This suggests that she tacitly participated in the shared preliminary knowledge structure that was publicly agreed upon. She also capitalizes on this construct in 5C.

In 5D, Yael is trapped by the challenge of a range including the expected value – a crucial part of the focus knowledge structure, and this pushes her to estimate the probability of 5D according to the probability of the simple event 0.25 (Yael 100). Later on, while the discussion continues in the class, she becomes convinced that the probability is high and marks D close to 1. However, we will see later that she did not *consolidate* this part of the focus structure, suggesting that she perhaps never even *constructed* it (Yael 115).

Yael marks 5E (N exactly 25 times) close to 1. It seems that she *recognized* in this task a relationship to a non-relevant part of the focus knowledge rather than to the relevant one. Specifically, 5E following just after 5D, she may have been led by the answer to 5D (which also refers to the expected value) that was still quite fragile for her, rather than by the fact that in repeated experiments the probability to obtain the same outcome exactly k times is very small (the preliminary knowledge structure), even in the case where k corresponds the expected value (the final part of the focus knowledge structure).

Rachel

At the end of the discussion on 5A, B and C, Rachel marks all events close to zero (A the closest, then B, and then C). So we can conclude that like Yael she agreed upon the shared knowledge concerning the preliminary knowledge structure as well as the first part of the focus knowledge structure.

During the discussion on 5D Rachel agrees with Guy (Rachel 110) after he and Omri co-explain why the probability of D is high (Guy 99, 101; Omri 104, 108); accordingly she marks D close to 1.

Rachel marks E lower than 0.5, in contrast to Yael, and the third girl in the group, Noam, who both mark it close to 1. We may assume that Rachel not only *constructed* the preliminary knowledge structure, but also *consolidates* it when using it in a difficult case (for the expected value itself) to answer 5E. I infer this from the fact that she *recognized* it, and *constructed* with it the knowledge that in repeated experiments the probability to obtain the same outcome **exactly** k times, where k corresponds the expected value (5E), must be smaller than the probability that the frequency of an outcome is in a specific range that includes the expected value. Nevertheless, she did not draw the correct conclusion that it must be close to zero.

I suggest that Rachel *constructs* her knowledge gradually but certainly: After having *recognized* in the discussion on 5C and 5D the problem of the (non-)inclusion of the expected value within the range, she undertakes all subsequent tasks dealing with this issue correctly (see, Rachel 161, and later her answers in the final quiz and interview). The issue that the probability for obtaining the same outcome **exactly** k times, is very low, is still fragile as we can see in 5E and later on (Rachel 138, 140).

Yael and Rachel in subsequent peer interaction on Task 6

Task 6 has been carried out in small groups. The three girls have a long discussion on the probability for the same outcome to repeat 1000 times. The preliminary knowledge structure is relevant in 6A (the outcome will be heads all 1000 times), and later in 6D (the outcome will never be heads). Following are some utterances, in which one can see how Yael convinced Rachel that such a probability is close to zero.

Yael marks A at 1/4 on the chance bar, but then immediately corrects herself:

Yael 132: *It's like when you throw a coin 10 times and you get 5 times heads and 5 times tails, you can't say that in 1000 that's 500 times heads and 500 times tails.*

She moves her mark close to zero, and later explains why the number of times counts:

Yael 137: *Yes it does! As you add more throws, your chances drop.*

Rachel understands that the event in 6A can hardly happen:

Rachel 138: *But if there are two sides, and you say yourself that there is not much chance it will come out 1000 times heads, then there are many times it will come out on tails. That's really what you are saying because the coin has only two sides.*

And later:

Rachel 140: *If you say that there were few heads, then many times, 1000, there was tails.*

But she still does not conclude that the probability is close to 0, and marks it at 1/4. Yael reacts:

Yael 143: *No, <u>all</u> the 1000 times you got heads, all the 1000 times?*

And later:

Yael 148: *1000 throws – the chance is low. It's not 1/4, it's much less. It is almost illogical that it should fall 1000 times on heads.*

It is worth noticing that these two utterances evidence Yael's construction of the preliminary knowledge structure.

Rachel seems to be convinced but still does not change her marking. Only after co-solving 6B and 6C, Yael returns in 6D to her explanation:

Yael 165: *Listen, could there be a case where all 1000 throws it came out only heads?*

She passes the eraser to Rachel. Rachel erases and corrects and puts a mark close to 0. She is not very active but seems willing, quite convinced, as can be seen from her answer in 6D and in the following tasks.

In 6D (The outcome will never be heads) all three girls declare together: *It's exactly like A.* They mark it at the same place as A, close to zero.

6B: The outcome will be heads between 450 and 550 times.

Noam 159: *That's at the half!*

Yael 160: *No, there is a much greater chance, it's what Guy explained.*

Rachel 161: *Right, she [Yael] is right.*

The three of them mark B close to 1.

Again it seems clear that Yael constructed that the part of the focus knowledge structure that concerns the probability that the frequency will be in a range that includes the expected value, presumably when they worked on question 5D.

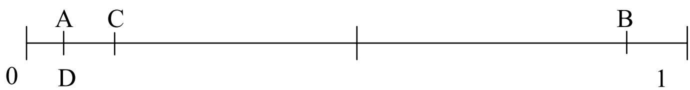
6C: The outcome will be heads between 850 and 950 times

Noam 162: *A little before A!*

Yael 163: *A little before? A "little" after!*

Noam 164: *Yes, about here. !*

The three of them mark the events on the chance bar at the same places. They mark C after A and D at about 0.15.



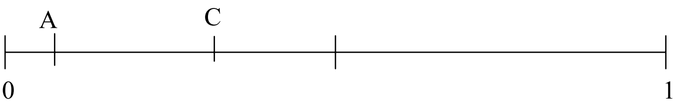
Further tasks

The girls did tasks 7 and 11 as homework. Thus their work can be evaluated according to their worksheets only. In 7 they all did approximately the same: They marked event A (heads between 450 and 550 times) close to 1 and event B (heads exactly 500 times) close to zero without any explanation.

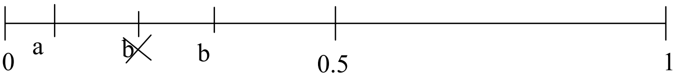
In task 11, Yael and Rachel wrote that Shira is right. Yael's explanation is: *Shira is right, as at average we will get 50 times an even number, but there is very little chance that it will be* <u>*exactly*</u> *50.* Rachel's explanation is: *I think that Shira is the closest to be right as there are not many chances that the die will fall* <u>*exactly*</u> *half of the times on an even number but I personally would have marked a little bit closer to half.*

Yael and Rachel's answers were quite similar. It seems that both of them *constructed* the focus knowledge structure. But our conclusions may be somewhat different if we also look at their final quiz and interview.

In Q3, in the final quiz, Yael correctly marked A (6 exactly 200 times out of 1200) close to zero, but marked C (6 between 100 and 300 times) around 1/3.



She acted similarly in T3 in the interview: She marked A (1 exactly 450 times) close to zero but B (1 between 400 and 500 times) around 1/3. From her worksheet it can be seen that she hesitated as she marked B first closer to A and then erased it, and moved her mark to the right.



Rachel answered the final quiz question correctly. She also acted in the interview correctly and similarly to the way she acted at the end of the activity in tasks 6B and 11. She marked A in the final quiz close to zero and C close to 1. She marked A in the interview close to zero and B close to 1.

Summary of Yael's and Rachel's actions

In Task 5 Yael is not very active, but from her marks on the chance bar and from her explanations to Rachel in Task 6, we may conclude that she had *constructed* the preliminary knowledge structure. This construction was not fully *recognized* in 5D and 5E, where the expected value is involved (probability that the frequency will be in a range that includes this value, or that it will correspond the expected value exactly). In these cases her actions are not systematic. This knowledge seems to be constructed by Yael in the four tasks of the activity (5, 6, 7 and 11) but was not *consolidated* at all as can be concluded from her responses in the final quiz and the interview.

Rachel, in contrast to Yael, did not appear to *construct* the knowledge in the class discussion in task 5, but her discussion with Yael during the group work on task 6, may have acted as a catalyst for this construction. It is very typical for Rachel that constructing the structure of knowledge went together with an immediate consolidation, which can be seen in her responses in the final quiz and the interview.

DISCUSSION

Researchers and theoreticians of learning (including learning in mathematics) have traditionally tried to find general features characterizing large populations (age groups, high level performers, experts, etc.). These attempts delineate an inclusion. Such an *inclusion* approach always conflicts with thorough and fine-grained analyses of empirical data concerning specific learning features of the various individuals in the community. It opposes *diversity*.

During the past decade, theories and research methodologies concerning the ways in which learning characteristics of various individuals should be observed and analyzed, have undergone deep changes: “Subjects” interviewed in laboratory conditions have been replaced by observations of (groups of) people in natural contexts, in various social settings (ensembles). Clearly, as the number of students in the ensemble increases, the difficulty to follow a single student becomes bigger and the information that a researcher is able to retrieve about the learning processes of the single student decreases. Noam, the third girl in the group with Yael and Rachel is a case in point – we have very little information on her.

The complexity of data collected on ensembles of students in “natural” settings can be enormous. The data presented in this paper are especially compound, because we started to follow the students in a whole class discussion (Task 5), moved to group work (Task 6), then to home work (Tasks 7 and 11), where they work separately, and eventually to the final quiz and the interview, which were also taken individually but in situations with very different risks for the students.

An second difficulty we face is the fact that we chose to investigate the *constructing* of knowledge of rather high complexity: First the difference between two connected probabilities: the probability of a specific outcome of a single event and the probability that the frequency of this same outcome in a repeated event has a specific value or is in a specific range. And then the idea that the probability that the frequency of an outcome (in a repeated event) is in a specific range of given length is large if the range includes the expected value, and small if the range is far from the expected value.

Moreover, a relatively short and interrupted time was allotted for this purpose: The tasks 5, 6, 7 and 11 that appeared in the second of five activities and the final quiz and interview that were carried out after the fifth activity.

In the first task (Task 5), which was discussed by the class as a whole, we could only assume whether and what the two girls had *constructed* while the constructing process of the class's shared knowledge took place. The second task (Task 6) was carried out by a group of three of which the two girls were the more active ones. From observing the group work, we obtained more detailed information on the girls' constructions, and were also able to make our previous assumptions concerning their *construction* (or not) in Task 5 more reliable. The homework tasks (7 and 11), which, was done individually and differently provided some information on the girls' knowledge structures immediately after the learning episode; we thus had some more information on what was constructed by each of the two girls (or not). Fortunately, we also had the final quiz and the interview and were able to see not only what was constructed but also what was *consolidated*. We note that knowledge may be constructed but remain available only for a short while; in a later stage the student may not recognize it as an already existing structure and thus not build-with it, and possibly not even be able to reconstruct it. This means that no consolidation of this short-term construction has occurred.

From an epistemological point of view, two constructions were involved in the short flow along the four tasks of Activity 2: the preliminary construction and the focus construction.

We were able to see that Yael, in spite of her ability to explain the preliminary construction to Rachel and in spite of her correct responses to some of the questions relating to the focus construction, did not appear to have constructed the focus construction. Thus her preliminary construction was not nested in any additional construction.

Rachel, in contrast, was able to consolidate the preliminary construction, to recognize the resulting knowledge structures during her focus construction, and thus her preliminary knowledge structure became nested in her focus knowledge structure.

In conclusion, I want to emphasize that even in such a short flow of constructing and consolidating actions, during which social and other contexts kept changing, it was possible to use the *inclusion* of the RBC model in order to obtain significant insight into two individual students' constructions of knowledge, enough insight to observe the *diversity* inherent in the differences between the two students' processes of abstraction.

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References

Cobb, P. (1998). Analyzing the mathematical learning of the classroom community: the case of statistical data analysis. In A. Olivier and K. Newstead (Eds.), *Proceedings of the 22nd Conference of the International Group for the Psychology of Mathematics Education,* Vol. 1 (pp. 33–48). Stellenbosch, South Africa: University of Stellenbosch.

Dreyfus, T., Hershkowitz, R. & Schwarz, B. B. (2001a). Abstraction in context II: The case of peer interaction. *Cognitive Science Quarterly 1*, 307-358.

Dreyfus, T., Hershkowitz, R., & Schwarz, B. B. (2001b). The construction of abstract knowledge in interaction. In M. van den Heuvel-Panhuizen (Ed.), *Proceedings of the 25th International Conference for the Psychology of Mathematics Education,* Vol. 2 (pp. 377 - 384). Utrecht, The Netherlands: University of Utrecht.

Granot, N. (1998). Unit of analysis in transit: From the individual's knowledge to the ensemble process. *Mind, Culture and Activity 5* (1), 42-66.

Hershkowitz, R. (1999). Where in shared knowledge is the individual knowledge hidden? In O. Zaslavsky (Ed.), *Proceedings of the 23rd Conference of the International Group for the Psychology of Mathematics Education,* Vol. 1 (pp. 9 – 24). Haifa, Israel: Technion – Israel Institute of Technology.

Hershkowitz, R. & Schwarz, B. B. (1999). Reflective processes in a technology-based mathematics classroom. *Cognition and Instruction* 17 (1), 65-91.

Hershkowitz, R., Schwarz, B. B. & Dreyfus, T. (2001). Abstraction in context: Epistemic actions. *Journal for Research in Mathematics Education 22*, 195-222.

Kieran, C., & Dreyfus, T. (1998). Collaborative versus individual problem solving: Entering another's universe of thought. In A. Olivier and K. Newstead (Eds.), *Proceedings of the 22nd Conference of the International Group for the Psychology of Mathematics Education,* Vol. 3. (pp. 112-119). Stellenbosch, South Africa: University of Stellenbosch.

Pontecorvo, C. & Girardet, H. (1993). Arguing and reasoning in understanding historical topics. *Cognition and Instruction 11,* 365-395.

Schwarz, B. B., Dreyfus, T., Hadas, N. & Hershkowitz, R. (2004). Teacher Guidance of Knowledge Construction. In these *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education.*

Shtein, H. (2003). Changes in students' concept of equation and solution in a technological learning environment. MA thesis submitted to the Weizmann Institute of Science, Rehovot, Israel [in Hebrew].

Tabach, M. (2001). *Knowledge Development of a Pair of Students: Beginning Algebra in an Interactive Environment.* MA thesis submitted to the Weizmann Institute of Science, Rehovot, Israel [in Hebrew].