“WHY DOESN’T IT START FROM THE ORIGIN?”: HEARING THE COGNITIVE VOICE OF SIGNS

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Starting from a situated cognition perspective, this paper reports on the activity of 9th grade students who are interpreting the shape of a graph arising from the motion of a bouncing ball. In an unfamiliar context, informed by previous knowledge of similar experiments, the obstacle of understanding why the graph does not start from the origin is overcome through an interplay between different signs.

THE IDEA OF A *SITUATED COGNITION*

> “When we deliberately and effortfully think, presumably muscles come into play. Primary among these are the speech muscles; for thought, as John B. Watson claimed, is primarily incipient speech. Thinking aloud is just uninhibited thinking. Other muscles enter the thought process too, as Watson appreciated, notably in the case of the artist or acrobat who plans his moves with incipient rehearsals of muscular involvement, or the engineer, who simulates in his muscles the lay of the land or the distribution of stresses in what he means to build. The artist, engineer, and acrobat are poor at putting their thoughts into words, for they were thinking with nonverbal muscles” (Quine, p.88).

The quotation above sheds light on the important role of body in the act of thinking; moreover, it stresses that this is not merely a matter of thought in itself. I argue that such a viewpoint holds just as for an *initial* phase of activation of thinking, as for the *constitutive* phase of its development; since this is true for all the human activities of thinking, it is significant for learning in particular. We have to bear in mind all the “ingredients” that constitute thought such as imagination, perception, motor-sensory receptivity, muscular activity, and brain information processing. Besides, thought is also matter of culture in a wide sense: learners’ expectations, motivations, tasks, goals, previous knowledge, etc. Although each of those aspects is quite meaningful in its own right, none of them in isolation can fully tell the story of a student conceptualisation. Therefore it is necessary to speak not simply of students’ cognition, but rather of what I call *cognition in context* or *situated cognition* (for a study within this perspective, see Watson, 1998). Since the reference discipline is Mathematics Education, *situated mathematical cognition* will be the general subject matter of my research.

SHAPING A CONSISTENT THEORETICAL FRAMEWORK

Recent developments and ongoing studies related to the psychology of mathematics highlighted biological and neurological constraints, cognitive mechanisms, and cultural roots affecting mathematical knowledge (see e.g. Butterworth, 1999; Dehaene, 1997; Houdé & Tzourio-Mazoyer, 2003; Lakoff & Nùñez, 2000). At present these new trends are not to be ignored because they clarify the central issue of the primary sources of mathematical understanding. More and more attention is being drawn to the role of perceptuo-motor activity (i.e., bodily actions, gestures, manipulation of materials or artefacts, acts of drawing, and so forth) in the learning of mathematics:

> "While modulated by shifts of attention, awareness, and emotional states, understanding and thinking are perceptuo-motor activities; furthermore, these activities are bodily distributed across different areas of perception and motor action based in part, on how we have learned and used the subject itself. […] the understanding of a mathematical concept rather than having a definitional essence, spans diverse perceptuo-motor activities which become more or less significant depending on the circumstances" (Nemirovsky, 2003, p.108).

This claim meets the idea of situated cognition. Furthermore, these perceptuo-motor activities are consonant with the semiotic "key elements in the organization of mental processes as they are used to reflect and objectify ideas in the course of the individuals' activities" (Radford, 2003, p.125): words, gestures, artefacts, drawings, and so on. As far as my research is concerned, the analysis has been focused on students' gesturing and linguistic productions, as observable constitutive components of the mental activity, by following the belief, to use famous words, that: "gestures, together with language, *help constitute thought*" (McNeill, 1992, p.245; emphasis in the original). My sense is that until recent years, gestures and language have often been taken into account in fragmented fashion, each in its own way, forgetting the richness of an interplay among them or between all the semiotic resources; but lately strong evidence has been published for their significance in constructing meanings in context, even in interaction with technology (see e.g. Alibali *et al*., 2000; Arzarello & Robutti, forthcoming; Nemirovsky *et al*., 1998).

A search for coherence: more signs

Among the contemporary cognitive oriented approaches, the theory of *embodied cognition* (Lakoff & Nùñez, 2000) has been for me the most fascinating one in that it investigates where mathematical concepts come from and examines the ways in which the embodied mind brings mathematics into being. Early interests were on studying the role of *conceptual metaphors* (the "fundamental cognitive mechanisms which project the inferential structure of a *source domain* onto a *target domain*"; Nùñez, 2000, p.9; emphasis in the original) in relation to mathematics learning. Later, metaphors came to be seen as vehicles of knowledge from the perspective of a social construction of meanings (Ferrara, 2003). On the other hand, we have analysed the mediation-role of technology in activities in which 14 years old students were asked to interpret on a calculator some graphs obtained from physical motion experiments in front of a sensor (Ferrara & Robutti, 2002). But soon, the question of a relationship between metaphorical thinking and the use of technology arose. Gestures and perceptuo-motor activities cannot be forgot nor understood without the means and tools that they put in motion. Many more *signs* need to be looked at to have a framework *consistent* with the idea of a situated cognition.

Signs and symbols are used in the literature sometimes with different meanings, and sometimes as synonymous. It is then important to make explicit how we use such terms in this study. With *signs*, I refer to what Radford (2002) calls "*semiotic means of objectification* – e.g. objects, artifacts, linguistic devices and signs that are intentionally used by individuals in social processes of meaning production in order to achieve a stable form of awareness, to make apparent their intentions and to carry out their actions" (p.15). Gestures will be part of signs, together with particular words, conceptual metaphors and blends, results of perceptuo-motor activities, keys on the technological artefacts, mathematical and general objects (like graphs, number tables, bouncing balls, etc.). Instead, with *symbol* I mean *the final widest and highest stage in the life of a sign*: that specific kind of sign embedding a conflation of the signified and the sign, in the present case a *fusion* (see Nemirovsky *et al*., 1998) of the physical phenomenon of motion and its mathematical graphical representation.

Within this perspective, I will discuss the mathematical cognition of some students (9^{th} grade) while striving to make sense of a position-time graph resulting from a bouncing ball. Roughly speaking, the paper arises from an attempt of *hearing the cognitive voice of signs (gestures, speech, artefacts, metaphors and so on) in such a situation*, trying to survey their role and interplay in making something non-palpable and unperceivable for students at the beginning (the shape of the motion graph) palpable and perceivable at the end.

THE CONTEXT

The experiment: methodology. The data comes from a long-term teaching experiment carried out a couple of years ago and part of a research project still in progress. The core activities involved grade 9 students in approaching the concept of function as model of a physical movement described through graphing (Ferrara & Robutti, 2002; Arzarello & Robutti, forthcoming). Two teachers, one of mathematics and one of physics, were active in the classroom and collaborated in designing the experiment sequence. In each activity the students worked in small groups (three to four people), then participated in a class discussion led by a researcher present in the classroom, and aimed at sharing and comparing the different solutions. We collected data for the analyses in the form of students' written notes, worksheets completed by the groups of students, and transcriptions from video-recordings of both, group and class discussions.

The activity. The activity this paper reports on is the fourth of a series. Their focus was on the construction of a model starting from physical motion (a second phase was centred on the inverse passage: from models to motions). In previous sessions, the students were asked to move in front of a CBR (a motion detector) to perform different kinds of motion: uniform (back and forth), accelerated and periodic. A red line on the floor marked the point where they had to stop or change direction. The last session differed from these in that the students used a toy object: a bouncing ball that groups dropped under the motion sensor. As the ball bounces, the students observed the position-time graph built in real time on the screen of a symbolic-graphic calculator (TI-92 Plus) linked to the sonar. After gathering data (stored in calculator memory) we provided each group with a worksheet encompassing three steps: first, the students explained in natural language the motion of the ball and the graph (e.g. "Describe the kind of motion the ball made", "Describe how space changes with respect to time (increases, decreases, etc.)"). Second, they were asked to interpret *qualitatively* the shape of the graph (e.g. "Analyse the graph. Is it like a straight line? Is it like a curve? Does the curve increase? Does the curve decrease?"). Finally, they had to interpret *quantitatively* the graph by calculating the slope of the curve at different points (e.g. "Consider the ratio m = $(s_2 - s_1)/(t_2 - t_1)$ and use it to describe mathematically the graph of your motion"[1]).

At the time of the experiment, in the first months of the school year and of high school, the students had not developed formalised knowledge in terms of graphs or functions. From an instructional view, the activities were designed to let the students pass gradually from an intuitive stage, starting from verbalising the experiment through natural language, to a more *conceptual* one. Because of space constraints a brief excerpt from the activity of one small group of students will be considered. Despite the briefness, it is significant for the different signs coming into play; I will strive to hear and understand through the analysis what their voice is saying.

DISCUSSION

The CBR was raised around 2 *m* from the ground and the ball was dropped under it. Only the first part of the graph (Figure 1) until 6 *s,* during which the bounces of the ball can easily be noted, is of interest; after 6 *s* the ball drifted away from the sensor range. The students are discussing the questions about the shape of the graph: Is it like a line? Is it like a curve? Does the curve increase? Does the curve decrease?".

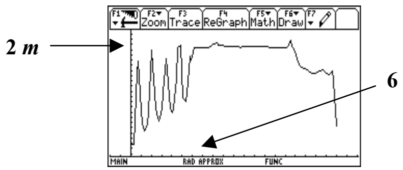

Figure 1

Soon a problem arises for Fabio and Giulia: the graph *does not* start from the origin. The *schema* (Radford, forthcoming) they have in mind differs from what the calculator screen shows. The previous artefact-mediated experiments, in particular the periodic motion (see the pronoun "we", #127), condition the students' expectations for the graph[2]. The schema needs to be re-thought of. But: *how?*

122. Giulia: *Why doesn't it start from the origin?*

123.	Fabio:	*When it arrives to this point here* [he is placing the cursor on an inferior extremity of the graph] *it is when the ball…*
124.	Giulia:	*Why doesn't it arrive on the horizontal axis?*
125.	Fabio:	*Here* [he is again placing the cursor on an inferior extremity of the graph] *it is as the ball, when it is near the CBR*
126.	Filippo:	*Yeah*
127.	Giulia:	*Hum, because we…*
128.	Fabio:	*Here it is when it* [the ball] *is near the CBR*
129.	Giulia:	*No*
130.	Fabio:	*Instead here* [he is moving the cursor on a superior extremity] *it is when…*
131.	Giulia:	*Here* [she is pointing to one of the superior extremity] *it is when it* [the ball] *is on the ground*
132.	Fabio:	*Here, when it is on the ground, just this point* [he is placing the cursor on the same superior extremity]
133.	Giulia:	*Yeah*
134.	Fabio:	*When it is on the ground* [he is moving the cursor on the next inferior extremity]
135.	Giulia:	*Whereas there, it is when it is near the CBR…*
136.	Fabio:	*And when it is near, when it approaches the CBR, the maximum point in which it approaches the CBR is here* [he is pointing to the inferior extremity of the graph where the cursor is located]

Not only the graph does not start from the origin but, in addition, it does not arrive on the horizontal axis (the physical ground, the floor, does not correspond to the 'mathematical ground'!). This requires a reflection both on the spatial origin of motion and on the position of the sonar (raised vertically, with the ball falling along a vertical trajectory). The point of view needs to be shifted to accommodate the ball as subject of motion. The passage happens with the help of a specific sign: the *Trace* key activated on the calculator displays a moving cursor on the graph, with the numerical coordinates corresponding to the point the cursor is at. Such a sign prompts a first exploration of the graph, rich of *deictic* and *locative* words (Radford, forthcoming): "here", "there", "this point"; "near the CBR", "on the ground". Usually these words are matched with physical body gestures; indeed, in the present case the *cursor* acquires the mediation function of an *index*, in place of usual *deictic/indexical* gestures (#123, 125, 130, 132, 134). Technology is working *with* the students, not *for* them: the instrument provides them with an inherent function (the tracing function) used through a physical tool (the cursor) to make accessible the motion experiment in the graph. The students already think of the graph in terms of motion (the pronoun "it" is used to speak of the curve 'starting', 'arriving', etc.); however, their interpretation remains at a *local* level, in terms of different spatial positions of the ball during its movement (this is an *operational* way to see the graph), as pointed out by the *timed* sentences (observe the pervasive presence of the temporal adverb "when"). The beginning of a more global conceptualisation of the graph is marked by their discovery that the maximum positions reached by the ball in motion correspond to the inferior extremities on the graph. A step more is made in #136, where a gesture substitutes the cursor.

137. Giulia: *Well, a curved line represents the graph…*
138. Fabio: *A curved line*
139. Giulia: *A curve that doesn't start from the origin*
140. Fabio: *Wait, say, let's see, hum…*
141. Giulia: *It is a curved line that has always the same… maximum point*
142. Fabio: *That… when…*
143. Giulia: *…but that varies as minimum point*
144. Fabio: *When the curve goes up, when it goes up* [he is sketching a parabolic slope in the air with his right hand: his index finger, pointing to imaginary physical locations in front of his body, follows this going up trajectory, from the bottom-left to the top-right]*, it indicates that the ball goes down towards the ground* [he is lowering his hand in a vertical direction, miming the ball motion while falling], [he fast raises his hand] *when it* [the curve] *goes down* [he is lowering his raised hand, reproducing with the index finger the previous slope but in the opposite versus]*, the ball* [he is raising his hand again in a vertical direction, miming the ball motion while bouncing up]...
145. Giulia: *It indicates that the ball goes up*

A second interpretation of the shape of the graph in terms of maximum and minimum points begins[3]. This is a *natural* resource for the students, so that they seem to have *embodied* the extreme spatial positions of the ball on the extremities of the graph (#141, 143). The students are now at a *global* level, in which the graph is considered in a *structural* way. Let us focus on the last two lines: Fabio shows of *knowing* and *acting* at once (#144): he knows the graph in terms of the motion of the ball, and he acts to reproduce both the shape of the curve and the motion itself. The schema has definitely been re-thought of: Fabio is able to enact the whole experiment and to *recognize* it *in* the graph, and Giulia shares this knowledge (#145). Graph and motion are not longer distinguished; in contrast they are *fused* together as marked by the use of verbs of motion, "to go up" and "to go down", for talking about the graph ("the curve", or "it" in the sentences) too. This process may be interpreted as the construction of a *blended space* (Lakoff & Nùñez, 2000), in which the features of the two domains of the *Graph is a Moving Object metaphor* (see Ferrara, 2003) are merged in a new unique sign: the motion-graph. On the other hand, this cognitive behaviour is also embedded in a convergence of Fabio's *iconic* gestures (Mc Neill, 1992), which are of two kinds. Informed by current trends in the study of gestures (see e.g. Alibali *et al*., 2000; Arzarello & Robutti, forthcoming; Edwards, 2003), I refer to them as follows: when Fabio's hand traces the shape of a piece of the curve, the gesture is *iconic-representational* (Arzarello & Robutti, forthcoming), standing for the graphical representation; when its movement enacts the motion of the ball, it is *iconic-physical* (Edwards, 2003), since it represents the physical phenomenon. The graph becomes something to think with, the motion something to enact. But, there is more information here: the transition from the operational (local) to the structural (global) mode of thinking of the graph mirrors the occurrence of a process of *reification*, which, through the birth of the Graph is a Moving Object metaphor, marks the beginning of the conceptualisation (for a study about the role of metaphors in constructing new concepts, see Sfard, 1994). As a consequence, rather than being simply the result of a conflation of the two existing domains of motion (the physical phenomenon) and graph (the abstract sign), *the blend arising from the metaphor is what brings the abstract graph-sign into existence as a symbol*. Similarly, the two Fabio's gestures are so coordinated with speech and among them and iterated during the explanation that, recalling the distinction by Peirce (1955) between three different kinds of signs (index, icon and symbol), I think it is possible to speak of them as gestures with a *symbolic* characterisation.

FINAL REMARKS

The analysis reported in this paper points out the need of further investigations: reification, blends, metaphors, knowledge objectification, special kinds of gestures or actions seem to be due to a *basic and inherent cognitive mechanism* activated in constructing a meaning, in grasping a concept, in conceiving a sign as symbol. On the other hand, reasoning in abstract terms or on abstract entities (such as mathematical objects, e.g. a graph) is a complex activity that requires drawing attention to the deep network of interactions of the students with the environment they are acting upon. Although it is not yet clear the fundamental mechanism entering the scene (and this remains an open problem), support may come from our neuro-biological structure. Recent neuroscience studies, for example, shed light on the representational dynamic of the brain as *non*-symbolic but as a type of self-organisation, in which body action plays a crucial role (Gallese, 2003). Results of this kind are to be regarded with a special eye, because they may provide us with productive answers.

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1. t_1 and t_2 are two subsequent time data, s_1 and s_2 are the two corresponding position data. A table on the calculator provides the students with the numerical values.
2. In previous motions, the graph starts from the origin: in fact, the student moving approximately begins his/her run at time t=0 where the CBR can gathers data (0.5 m). And at that time he/she has not yet covered any space.
3. As a consequence of *pixel definition* of the screen, there is not a perfect match between the plane locations of the cursor on the inferior/superior extremities and the maximum/minimum points. But this is not a problem, because a qualitative interpretation is enough for the students.

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