THE USE OF DIAGRAMS IN SOLVING NON ROUTINE PROBLEMS

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This paper explores the role of diagrams in a specific problem solving process. Two types of tests were administered to 194, 12 year old students, each of which consisted of six non-routine problems that could be solved with the use of a diagram. In Test A students were asked to respond to the problems in any way they whished whereas in Test B problems were accompanied by diagrams and students were asked to solve these problems with the use of the specific diagrams presented. The results revealed that there was no statistical significant difference between the two tests. The result also revealed that it was not the same group of students that were successful in the two tests.

INTRODUCTION

The mathematics education community has espoused the importance of developing children's problem solving skills (for example, National Council of Teachers of Mathematics, 2000; Shoenfeld, 1992). In the same vein, research in mathematics education discusses the importance of using multiple representations in the problem solving process (Lesh, Behr, & Post, 1987; English, 1996). Markmann (1999) interprets the term "representation" as the concept that includes the represented world, a representing world, a set of rules that map elements of the represented world to elements of the representing world, and a process that uses the information in the representing world. Diagrams are considered as one kind of such representations (Novick & Hurley, 2001). The represented world in this case is a description of a problem to be solved, while the representing world contains the spatial diagrams as an abstract form, along with their applicability conditions. Specifically, a diagram is a visual representation that displays information in a spatial layout (Diezmann & English, 2001). In problem solving a diagram can serve to represent the structure of a problem. Diagrams are considered structural representations, in which the surface details are not important and this is their main characteristic and differentiation from pictures and drawings (Veriki, 2002). Diagrams typically rely on conventions to depict both the components of the situation being represented and their organization. These conventions must be learned and understood before the diagrams can be understood and successfully used (Diezmann & English, 2001).

According to a number of researchers the ability to use diagrams is a powerful tool of mathematical thinking and problem solving, because they are used to simplify complex situations, they concretize abstract concepts, and they substitute easier perceptual inferences for more computationally intensive search processes and sentential deductive inferences (Novick & Hurley, 2001; Diezmann & English, 2001;

Kidman, 2002).This paper discusses a part of a larger study which aims to investigate the impact of diagrams in solving non-routine problems.

THEORETICAL BACKGROUND

Networks, Matrices and Hierarchies

The efficient use of a diagram depends on its suitability for a given situation. The appropriateness of a diagram depends on how well it represents the structure of the problem. Novic and Hurley (2001) propose three general–purpose diagrams that suit a range of problem situations, networks, matrices and hierarhies. These diagrams highlight structural commonalities across situations that are superficially very different. These diagrams are especially useful in elementary non-routine problems (Booth & Thomas, 2000; Diezmann & English, 2001). Networks consist of sets of nodes with one or more lines emanating from each node that link the nodes together. Networks do not have any predefined formal structure (Fig.1). Matrices use two dimensions to represent the relationships between two sets of information. A cell in the matrix denotes the intersection of value i on one variable and value j on the other variable (Fig.2). Hierarchies comprise diverging or converging paths among a series of points. A single node gives rise to at least two other nodes (Novick & Hurley, 2001; Diezmann & English, 2001) (Fig.3).

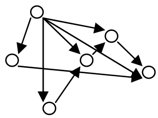

Figure 1: Networks

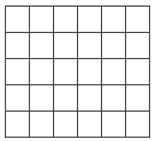

Figure 2: Matrices

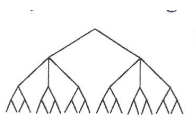

Figure 3: Hierarchies

In Networks the nodes specify values along a single variable. Any node may be linked to any other node and all of them have identical status. In Matrices the rows and columns specify values along two distinct variables. Values on the same dimension may not be linked. All the rows have identical status, as do all of the columns. In Hierarchies the nodes at a given level have identical status, but the nodes at different levels differ in status. While in Networks the links between the nodes may be unidirectional, in Matrices the links between the nodes are non directional, while in Hierarchies the links are directional (Novick & Hurley, 2001; Diezmann & English, 2001). Networks and Hierarchies may show paths connecting subsets of nodes, while matrices do not show subsets.

Similar diagrams are often used in solving non routine problems. Non routine problems are the problems that do not involve routine computations but the application of a certain strategy, in this case a diagram, is required in order to solve the problem (English, 1996). Non routine problems are considered more complicated and difficult than routine problems in which only the application of routine computations is involved in their solution (Shoenfeld, 1992). Research concerning the efficiency of the use of diagrams in solving non routine problems is often contradicting and thus inconclusive (Diezmann, & English, 2001; English, 1996; Booth & Thomas, 2000). Thus, the present study purports to throw some light to the contribution of the three diagrams in the solution of non-routine problems. More specifically, the study investigated two questions:

(a) Does the *presence* of diagrams in non-routine problems increase students' ability in solving them? and

(b) How does the presence of diagrams in non-routine problems affect students' reponses?

(c) Does the *use* of networks, hierarchies and matrices (diagrams) facilitate students in solving non-routine problems?

(d) Does the use of diagrams facilitate all students?

METHOD

To examine the impact of diagrams in the problem solving process we constructed two tests. Test A consisted of six problems. The problems used in the study were chosen based on three criteria. First, the problems were taken from previous pieces of research, which showed that children used diagrams similar to the ones that we intended to use in this study in order to solve them (Diezmann & English, 2001; Booth & Thomas, 2000). Second, students were familiar to similar problems since they appear in their mathematics textbooks, and third, the teachers' guide book suggests that these problems should be solved using similar diagrams with the ones we indebted to use in the study.

The first problem inquired about the distance between four trees. Specifically, students had to find out the distance between two trees while the distance of the other trees was given. In the second problem students had to find the step that the cleaner was standing when he started cleaning the windows. Problems 1 and 2 could both be solved with the use of Network diagrams, because the problems had identical status items (trees and steps) that may be linked with each other. In both problems it was important to show paths connecting the items.

The third problem concerned combinations $A \times B \times C$. Children were asked to find out all the possible combinations of two cards (Christmas or Easter cards), their colour (green or yellow) and their ribbon's colour (red or blue). The fourth problem asked children to find out the wining team among four teams. Problems 3 and 4 could be solved using the Hierarchy diagram because they involved items that were distinguished according to different levels. In addition, the links between the items were directional and only one route existed between any two items.

The fifth problem asked children to match each of the four friends with their own favourite kind of books according to some given information. The sixth problem was a combination problem $A \times B$ (four flavours of ice cream and three types of cones). These two problems could be solved using the Matrices diagram because both of them had two sets of items. The items within a set could not be combined amongst themselves (for example the 3 different cones). All possible combinations of the items across sets should be considered (cones and flavours).

Test A was administered to 194 Cypriot students in grade 6. Students were asked to solve the six problems (a1-a6) in any way they wished. Test B contained isomorphic problems, this means problems that had the same structure as those in the first test and was administered to students a week after the administration of Test A. Each problem in Test B (b1-b6) was accompanied by a diagram (bgi-bg6) that students were asked to use in order to solve the problems.

For the analysis and processing of the data collected, statistical analysis was conducted by using the computer software SPSS and CHIC (Bodin, Coutourier, & Gras, 2000). A t-test analysis was produced to examine the difference between students' achievement in the two tests. The statistical package CHIC produces three diagrams. The similarity diagram which represents groups of variables which are based on the similarity of students' responses to these variables. The implication graph which shows implications $A \Rightarrow B$. This means that success in question A implies success in question B. Finally the hierarchical tree which shows the implication between sets of variables. In this study we use only the first two.

RESULTS

In regard to the first question of this study the mean score of Test A was 14,36 (20) and the mean score of Test B was 14,55(20). The t-test analysis revealed that there was no statistical significant difference between students' achievement in the two tests ($M\alpha$-$M\beta$=-0,196, df=193, p=0,636).

In regard to the second question of the study, according to the similarity diagram (figure 4), students' responses to the problems of the two tests are separated except from the problem b1. The emergence of these two clusters of variables (a1,a2,a3,a5,a6) and (b2,b6,b3,b4,b5) show that students perceive the two tests as completely different tasks failing to realise that the two tests included isomorphic problems. The similarity diagram (Figure 4) shows that students have not realized the structural resemblance of the problems in the two tests. With the presence of the diagrams in Test B they encountered the problems as completely different tasks. It is not clear why b1 joined cluster a. It may be conjectured that the high level of difficulty of problems a1 and b1 caused this similarity between them.

The similarity diagram also shows statistically significant similarity at level 99% (Note: Similarities presented with bold lines are important at significant level 99%) only between the problems in test A. Particularly, there is statistically significant similarity between the variables a1-a3. The similarity between students' responses in problems a1 and a2 may be due to their similar structure. There is also a statistically significant similarity between the group of variables a1-a3, and the group of variables a5-a6. This can be explained in certain extend to their similar structure (a5 and a6, a1 and a2) and the level of difficulty of these non-routine problems. As it was pointed out earlier on, these problems are considered difficult and therefore are usually solved

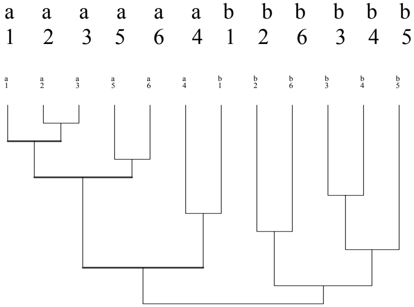

Figure 4: Similarity diagram of the responses to the problems in the two tests

by mathematically able students. On the contrary in Test B there does not appear to be any statistically significant similarity at level 99% between any of the problems, suggesting that the presence of each diagram in the problems had a determinative role to students' responses. One explanation that may be given is that the appearance of the diagrams was causing different behaviour to students resulting to different responses. Whereas in Test A success in the problems depended solemnly on students mathematical abilities, in Test B students also had to interpret and use the diagrams efficiently. This may have caused the lack of any similarity between students' responses in Test B.

The implication graph in figure 5 shows significant implicative relations between the problems in Test A. Specifically, it suggests that success in a1 implies success in a3. Therefore in Test A, a1 was the most difficult and a4 the easiest problem. While in Test A there is a hierarchy of difficulty, shown in the implication graph in figure 5, this hierarchy does not exist in Test B. This implies that with the presence of the diagram some problems became easier for some students, while other problems became more difficult. In addition, the implication graph shows that students who solved correctly a problem in Test A did not necessarily solved correctly the isomorphic problem in the Test B and reversely.

In regard to the third question whether the use of the diagrams facilitates students ability in solving non-routine problems, figure 6 gives some interesting information. The implication graph shows that students who used the networks in b1 and b2 also used the hierarchy in b3 and matrices in b6. However, this use does not imply the successful solution of the foremost mentioned problems.

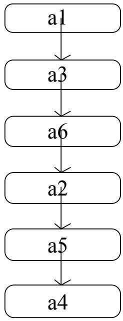

Figure 5: implication graph illustrating relations among the twelve variables, a1-a6 and b1-b6.

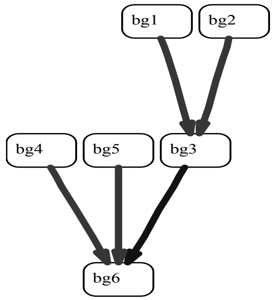

Figure 6: implication graph illustrating relations among the use of the diagrams bg1-bg6 and the correct answers to problems b1-b6.

In regard to the fourth question, Table 1 shows in detail the students' responses to the two tests.

Wrong Responses in Test A		Responses in Test B		Correct Responses in Test A		Responses in Test B	
A1	W:**112 (58%)**			A1	C: **82 (42%)**		
		B1	W: **84 (75%)**			B1	W: **41 (50%)**
		B1	C: **28 (25%)**			B1	C: **41 (50%)**
A2	W: **77 (40%)**			A2	C: **117 (60%)**		
		B2	W: **43 (56%)**			B2	W: **37 (32%)**
		B2	C: **34 (44%)**			B2	C: **80 (38%)**
A3	W: **97 (50%)**			A3	C: **97 (50%)**		
		B3	W: **57 (59%)**			B3	W: **33 (34%)**
		B3	C: **40 (41%)**			B3	C: **64 (66%)**
A4	W: **22 (11%)**			A4	C: **172(89%)**		
		B4	W: **12 (55%)**			B4	W: **23 (13%)**
		B4	C: **10 (45%)**			B4	C: **149 (87%)**
A5	W: **62 (32%)**			A5	C: **132(68%)**		
		B5	W: **27 (44%)**			B5	W: **41 (31%)**
		B5	C: **35 (66%)**			B5	C: **91 (69%)**
A6	W: **75 (39%)**			A6	C: **119 (61%)**		
		B6	W: **33 (44%)**			B6	W: **15 (13%)**
		B6	C: **42 (66%)**			B6	C: **104 (87%)**

Table 1: Pupils' responses to problems a1-a6 and their responses to the isomorphic problems b1-b6. (W: Wrong responses, C: Correct responses)

The important aspect of the Table 1 is that although these problems are considered difficult for primary school level and usually they are expected to be solved only by mathematically able students, the presence of the diagram in Test B has been very helpful for some students. However these students were not the same as those who solved the isomorphic problem in Test A. This is evident by the fact that a number of students that have solved the problem correctly in Test A they were not able to solve the isomorphic problem in Test B. Some students seem to have failed to see the representing structure in the diagram of the problem, while for other students the diagram proved to be very helpful.

DISCUSSION

The present study is within the framework of the ongoing discussion about the role of mathematical representations, and more specifically of diagrams, in the problem solving process. The results of the study suggest that the presence of the diagrams in Test B did not increase students' ability in solving the non routine problems. This is exemplified by many students' failure to "see through" the diagram the structure of the problem even though similar diagrams are used for the solution of these kinds of problems in their classroom.

The presence of each diagram had a determinative role in students' responses to each problem. While students' responses to Test A depended solemnly on students' mathematical ability, in Test B students also had to interpret and use the diagrams efficiently. It appears that not all students were able to do so. Success or failure to use the diagrams correctly might be due to a number of reasons. First, students' inability to interpret the diagram correctly and second, some students' lack of experience of solving problems with the presence of diagrams.

The results of the study show that the efficient use of a diagram did not imply the successful solution of a problem and reversely the successful solution of a problem did not imply the efficient use of the accompanied diagram. It can be argued that students perceived the problems with the accompanied diagrams as different tasks failing to perceive each diagram as an additional aid for the solution of the problems.

The results of the study show that the diagrams make the problems easier for some students while they make the problems more difficult for some other. For this reason, students who solved the problems in Test B were not the same students who solved the problems in Test A. It can be claimed that pupils with different visuo-spatial abilities (Booth & Thomas, 2000) responded differently in the tests.

Diagrammatic literacy is an essential component of students' mathematical development (NCTM, 2000). In order to use diagrams efficiently, students must develop the ability to translate the word problem into a diagrammatic representation and the ability to interpret a diagram in terms to a given word problem (Novick & Hurley, 2001; Diezmann & English, 2001). However, what is clearly suggested in this study is that this is a skill that needs to be developed and is not inherent to all students. A specific diagram does not have the same impact on all students. It is likely that the diagrams presented may be incompatible to some students' mental representations. Therefore, it is very important for students to be engaged in multiple representations in the problem solving process. Development of students' diagrammatic literacy may only be achieved though carefully designed instructional activities. These may turn diagrams into an effective tool for thinking.

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